# NEW INSIGHTS ON INTERMITTENCY AND CIRCULATION STATISTICS OBTAINED FROM A MASSIVE TURBULENCE SIMULATION DATABASE

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## EXECUTIVE SUMMARY

Substantial advances in both domain science and computing for fluid turbulence have continued in this work on Blue Waters. A recent focus is in the study of intermittency, which is mostly characterized by multipoint descriptors of turbulence structure, whose multifractal characteristics have in turn led to significant difficulties in analyses. However, the research team has found that the circulation of velocity around a closed contour or loop (equivalent to a two-dimensional area average of a vorticity component) has much simpler ("bifractal") properties. In particular, if all sides of a rectangular loop are within the inertial range of scale separations then the circulation depends, at least to a very close approximation, only on the size of the loop but not its shape. The study of circulation also demonstrates how a massive high-resolution turbulence database at high Reynolds numbers can provide much clearer answers than previously feasible in the literature.

## RESEARCH CHALLENGE

Since turbulence is characterized by fluctuations arising over a wide range of scales, many research strategies focused on fundamental understanding have been formulated in the context of a search for scale similarity (or departures therefrom). For example, moments of the instantaneous energy dissipation rate averaged over spatial regions of linear size varying over a substantial range provide a telling indicator of flow structure (Fig. 1) as well as playing an important role in corrections of classical similarity theory to account for the effects of intermittency [1]. However, researchers have also learned that [2] as a result of demanding resolution requirements [3], precise and reliable results are (whether experimentally or numerically) very difficult to obtain, which is especially the case for high-order statistics at high Reynolds numbers.

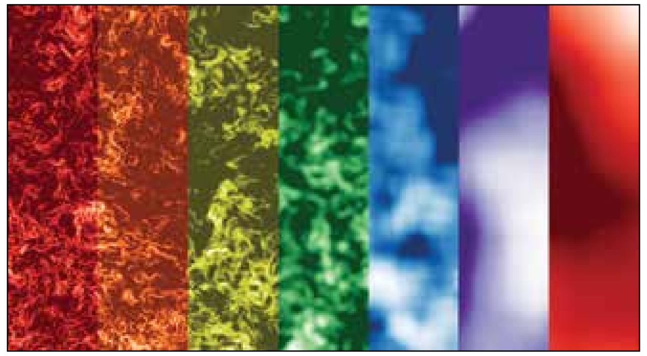

Figure 1: A mosaic of locally averaged slices of dissipation rate averaged locally over cubic subdomains of increasing scale size, from left to right, showing a change from wrinkled to smooth appearance.

In fluid dynamics, circulation is defined as the line integral of the velocity vector around a closed loop, or, equivalently, the integral of a vorticity vector component over the area enclosed. This concept is important in the occurrence of aerodynamic lift, oceanic transport, mantle convection inside the core of the Earth, and other contexts. In principle, the statistics of circulation will depend on both the size and shape of the loop, but a theory known since the 1990s [4] suggests that only the size matters, provided that length scales on every side are within the inertial range. A wide inertial range is necessary for this theory to be tested properly. As a result, it is not surprising that several studies in the past [5,6] were not conclusive. However, since computations using machines such as Blue Waters [7] have reached resolution levels some 4,000 times larger (in terms of total number of grid points) than the state of the art of the 1990s, there is reason for new optimism.

## METHODS & CODES

The research team integrated the Navier–Stokes equations over a large number of timesteps, using Fourier pseudo-spectral methods in space and finite differences in time. Circulation was computed via postprocessing of a substantial number of instantaneous snapshots of the velocity field saved during the simulations. While investigators can use either the line integral or area integral definitions as noted above, the area integration approach is more convenient. It also has the advantage of providing a more intuitive connection to the vorticity vector, which characterizes the tendency of local fluid elements to rotate on their own axes as a result of deformation by the turbulent flow. The required code development mainly resides in implementing a two-dimensional domain decomposition used to perform the simulations.

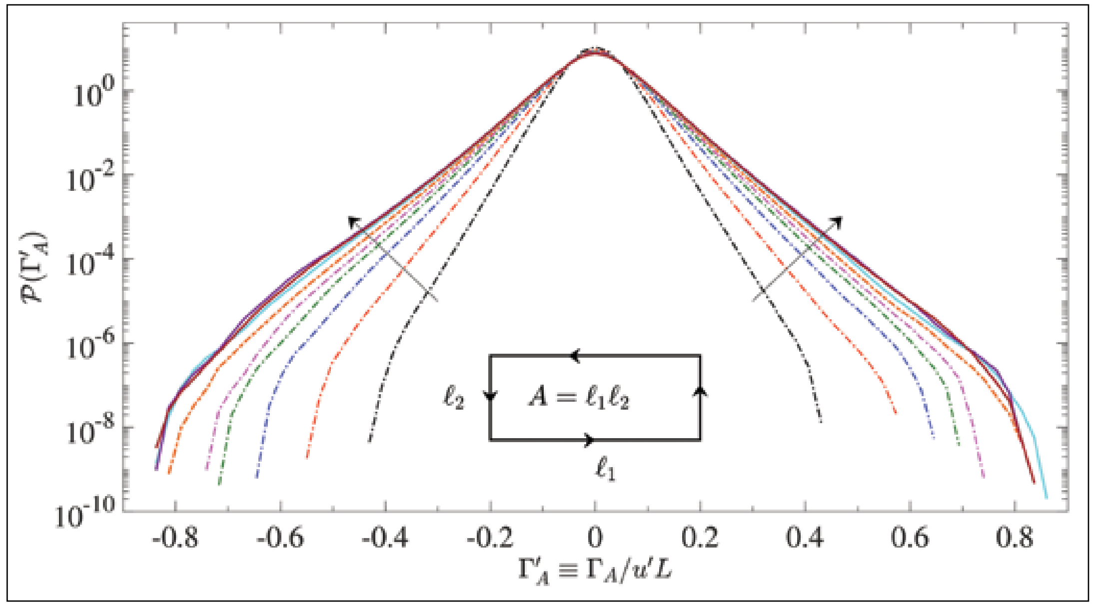

Figure 2: Probability density function of normalized circulation for loops of different aspect ratios and areas increasing in the direction of the arrows. Solid lines indicate loops with all sides in the inertial range but different shapes nearly coincide, in contrast to dashed lines for loops not wholly in the inertial range.

## RESULTS & IMPACT

In [8], the research team reported on a detailed study of the statistical properties of circulation using a large database in isotropic turbulence over a range of Reynolds numbers. The highest grid resolution analyzed involved over four billion grid points (16,384 in each of the three Cartesian coordinate directions). The team primarily studied circulation computed as area integrals of the vorticity over rectangular loops. The simulation results (Fig. 2) provide strong support for the theoretical prediction (noted above) that the statistics of circulation depend only on size but not the shape of the loop, provided the entire loop is all constrained within the inertial range. The probability density function of the circulation appears to scale in a manner consistent with the classical Kolmogorov 1941 hypotheses without significant effects of intermittency. Furthermore, the circulation is shown to exhibit, to excellent accuracy, a bifractal behavior at the higher Reynolds number considered: space-filling for low-order moments, following the paradigm of Kolmogorov 1941 [9,10], and a monofractal with a dimension of about two for higher orders. This change in character, occurring roughly at the fourth moment for the highest Reynolds number considered, is reminiscent of a phase transition encountered in other branches of physics.

The evidence obtained in this study points to a reduction in complexity when considering averaged vorticity over loops, which may provide a route to circumventing the spatial complexities involving velocity differences and gradients in turbulence [11]. This conclusion suggests that a great simplification, in principle, of the intermittency problem in three-dimensional turbulence may be possible when viewed through the lens of vorticity correlators in loop spaces. This great simplification may, in turn, have the impact of invigorating the use of similar loop formulations in tackling turbulence.

## WHY BLUE WATERS

The 8,192$^3$-grid resolution of the research team's production simulations requires access to a world-class machine such as Blue Waters. The machine capacity on Blue Waters has also proven sufficient to allow the team to obtain data at even higher resolution, *i.e.*, 12,288$^3$ and even 16,384$^3$ (although only for short periods of time), which is the highest known worldwide in the turbulence community. Indeed, 16,384$^3$ data, although short in time span, have been the focus of analysis presented in [8].

## PUBLICATIONS & DATA SETS

D. Buaria, A. Pumir, E. Bodenschatz, and P.K. Yeung, "Extreme velocity gradients in turbulent flows," *New J. Phys.*, vol. 21, p. 043004, 2019.

K. P. Iyer, J. Schumacher, K. R. Sreenivasan, and P. K. Yeung, "Scaling of locally averaged energy dissipation and enstrophy density in isotropic turbulence," *New J. Phys.*, vol. 21, p. 033016, 2019.

P. K. Yeung, "Extreme events, resolution, and onward to exascale computing," presented at the Prospectives on Turbulence Workshop, Texas A&M University, College Station, TX, U.S.A., Aug. 2018.

P. K. Yeung, "Massive computations in 3D turbulent flow: Science goals and an asynchronous algorithm towards the exascale," presented at Amer. Geophys. Union 100th Fall Meeting, Washington, DC, U.S.A., Dec. 12, 2018.

P. K. Yeung, "Extreme scale computing in turbulence: physics and algorithms," presented at an invited seminar at the Center for Environmental and Applied Mechanics, The Johns Hopkins University, Baltimore, MD, U.S.A;, Mar. 8, 2019.