Ab Initio Models of Solar Activity

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Solar Explosions: CMEs
GOAL: Understand Active Regions

• Magnetic fields are generated by dynamo action in solar convection zone
• Fields erupt through the visible solar surface to produce pores, sunspots and active regions
• New field interacts with existing field in the atmosphere to store and release magnetic energy which produces the explosions

Method: magneto-radiation-hydrodynamic simulations
Challenge

• Physics
  o Excitation and Ionization
  o Radiation energy transport
  o Turbulence

• Spatial & Temporal Range
  o DKIST will resolve 30 km
  o Convective structures 1-100 Mm
  o Surface convection – minutes, deep convection - days
Magneto-Hydrodynamic Equations

- Mass conservation
  \[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \]

- Momentum conservation
  \[ \frac{\partial (\rho \mathbf{u})}{\partial t} = -\nabla \cdot (\rho \mathbf{uu}) - \nabla P - \rho \mathbf{g} + \mathbf{J} \times \mathbf{B} - 2 \rho \mathbf{\Omega} \times \mathbf{u} - \nabla \cdot \mathbf{\tau}_{\text{visc}} \]

- Energy conservation
  \[ \frac{\partial e}{\partial t} = -\nabla \cdot (e \mathbf{u}) - P \nabla \cdot \mathbf{u} + Q_{\text{rad}} + Q_{\text{visc}} + \eta J^2 \]

- Induction equation & Ohms law
  \[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + (1/\varepsilon n_e) (\mathbf{J} \times \mathbf{B} - \nabla P_e), \]
Numerical Method

- Spatial differencing
  - 6th-order centered finite difference.
- Time advancement
  - 3rd order, Runga-Kutta
- Equation of state
  - tabular
  - including ionization
  - H, He + abundant elements
- Radiative transfer
  - 3D, LTE
  - 4 bin opacity distribution function
- Diffusion

\[
\frac{\partial f}{\partial t}_{\text{diffusion}} = \left( \frac{\partial}{\partial x_i} \right) \left( \nu \alpha \left( \frac{\partial f}{\partial x_j} \right) \right)
\]

\[
\alpha = \Delta^3 / \max \left( \left| \Delta f \right|_{-2,-1,0,+1,+2} \right)
\]

\[
v_i = c_1 \left( c_{\text{sound}}^2 + c_{A_\text{Alfven}}^2 \right)^{1/2} + c_2 |u_i| + c_3 \left[ (\Delta_3 u) < 0 \right] \Delta x_i
\]
6th order Finite Differences

\[ f'_{i+1/2,j,k} = \frac{a}{\Delta x} (f_{i,j,k} - f_{i+1,j,k}) + \frac{b}{\Delta x} (f_{i-1,j,k} - f_{i+2,j,k}) + \frac{c}{\Delta x} (f_{i-2,j,k} - f_{i+3,j,k}), \]

where

\[ c = \frac{3}{640}, \quad b = \frac{-1}{24} - 5c, \quad a = 1 - 3b + 5c. \]

5th order Interpolation

\[ f_{i+1/2,j,k} = a(f_{i,j,k} + f_{i+1,j,k}) + b(f_{i-1,j,k} + f_{i+2,j,k}) + c(f_{i-2,j,k} + f_{i+3,j,k}) \]

where

\[ c = \frac{3}{256}, \quad b = \frac{-25}{256}, \quad a = 0.5 - b - c \]
Key Challenge: Radiation Transport

• Radiation transport is inherently 3D & non-local. It couples distant regions → lots of communication. STAGGER uses long characteristics, filling the volume. Need to communicate volume data.

• Solution: restrict transfer calculation to only surface layers where it is important for the energy balance.

• Restrict number of frequencies (energies) and directions (rays).
Vertical and 4 angled rays
One through each surface cell
Angled rays rotate each time step,
  sweep out volume
Multigroup opacity and source function.
Bin frequencies according to opacity magnitude.
Use 4 bins, need 12 for precise agreement with observations.
Boundary Conditions

• Vertical:
  – Density: Top extrapolate lnρ. Bottom-inflows fix ρ, -outflows ρ\(\rightarrow\)\(<\ρ>\).
  – Velocity \(\rightarrow\) constant @ top, zero derivative @ bottom;
  – E=energy/mass Top: \(\rightarrow\) average value, Bottom: extrapolate \(<E>\) outflows, fix E inflows.

• B tends to potential field @ top,

\[\text{B advected by Inflows @ bottom (20 Mm) -- Weak (1 kG) or Strong (5 kG), minimally structured (horizontal, uniform, untwisted) magnetic field.}\]

Represents top of larger, rising flux concentration.

Imposed via specifying the horizontal electric field.
Simulations

• Variable is field strength and geometry (controlled by the convection, deeper \(\rightarrow\) larger).

• Project:
  ① Extend computational domain from 20 to 30 Mm depth so has larger convective cells and overlaps interior, global dynamo calculations.
  ② Use dynamo data \(\rightarrow\) spatially and temporally varying magnetic boundary condition.
Observed AR Flux Emergence: Vertical Field
Simulated Vertical B

Time 40.00 hrs
Tracking magnetic field lines: Rising Magnetic Loop
Summary

• Use BW 32-64K nodes to model AR formation by magneto-convection.
• Extending domain in depth and width to accommodate realistic solar AR.
• Provides synthetic data for improving & validating helioseismic inversions of magnetic regions.
• Provides synthetic data for analysis of observations from new solar telescopes: NST, Daniel K. Inouye Solar Telescope (DKIST, formerly ATST)
• Other parts of project await completion of extension to 30 Mm depth x 192 Mm width.