Two-Fluid Turbulence and Two-Fluid Dynamos in Molecular Clouds. Additionally: A New Paradigm for Computational Astrophysics.

By

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Why this project? Through HAWC+ polarimeter on SOFIA, NASA has made multi-million dollar investments in instrumentation to study magnetic fields in such plasmas.

This project provides the theoretical back-end for the matching observational program.

How stars form is a major astrophysical problem. All star-formation takes place in turbulent partially ionized plasmas. This project studies such turbulent plasmas.

Why Blue Waters?:- The project calls for 3D billion+ zone simulations of two-fluid turbulence; studying the evolution and growth of magnetic fields in such a partially ionized plasma. The simulations are extremely time-consuming but there is no other way of gaining insight except via these simulations.

Blue Waters is the only university-accessible platform that can support such simulations.

The Value-Add for NSF/NCSA/XSEDE: We have worked out a new paradigm for Computational Astrophysical MHD on Geodesic meshes.

Been able to show that this new paradigm also scales spectacularly well on Blue Waters.

Our work has also introduced new CoArray Fortran capabilities in the GNU compiler suite which is available on all XSEDE platforms and also Blue Waters.

(CAF and MPI-3 on BW are comparable and vastly superior to MPI-2 on BW.)

Outline

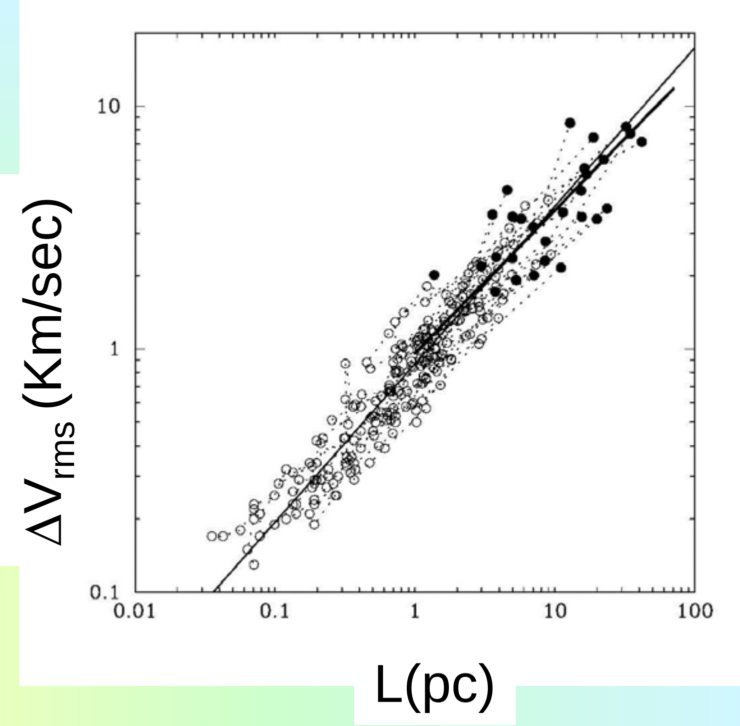
- Introduction – Star Formation, Molecular Clouds
- Wave Propagation in Partially Ionized Systems
- Our Simulations of Two-Fluid Turbulence
- Our Simulations of Two-Fluid Dynamo
- Geomesh MHD – A new paradigm for Computational Astrophysics
- Conclusions

I) Giant Molecular clouds

20 to 50 pc across; $n \sim 10^3 - 10^5$ #/cm^3

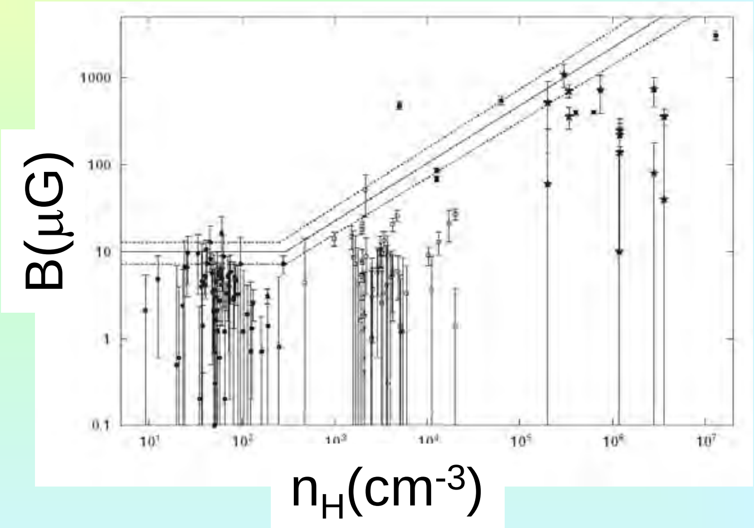
Highly magnetized $B \sim 30\mu G$. $V_A > C_s$

They have high Mach number turbulence. Mach 5 to 15 not unreasonable.



Linewidth-size relationship, Larson (1981) : $\Delta V_{rms} \propto L^{0.3\text{-}0.5}$

B-field increases with density (Crutcher et al 2010):



Partially ionized plasma

Ions and **neutrals** -- reasonably well-coupled on large scales – decoupled on smaller scales

Newton's Laws for Partially-Ionized Fluids

$$\rho_i\left(\frac{\partial \mathbf{v}_i}{\partial t}+\left(\mathbf{v}_i\cdot\nabla\right)\mathbf{v}_i\right)+\nabla P_i+\rho_i\nabla\Phi+\frac{1}{4\pi}\mathbf{B}\times\left(\nabla\times\mathbf{B}\right)=-\alpha\rho_n\rho_i\left(\mathbf{v}_i-\mathbf{v}_n\right)$$

$$\rho_n\left(\frac{\partial \mathbf{v}_n}{\partial t}+\left(\mathbf{v}_n\cdot\nabla\right)\mathbf{v}_n\right)+\nabla P_n+\rho_n\nabla\Phi=-\alpha\rho_n\rho_i\left(\mathbf{v}_n-\mathbf{v}_i\right)$$

V.V. Imp.

$L_{AD} = \mathrm{V}_A/\alpha\rho_i$ ~ 0.01 -- 0.05 pc for fiducial parameters

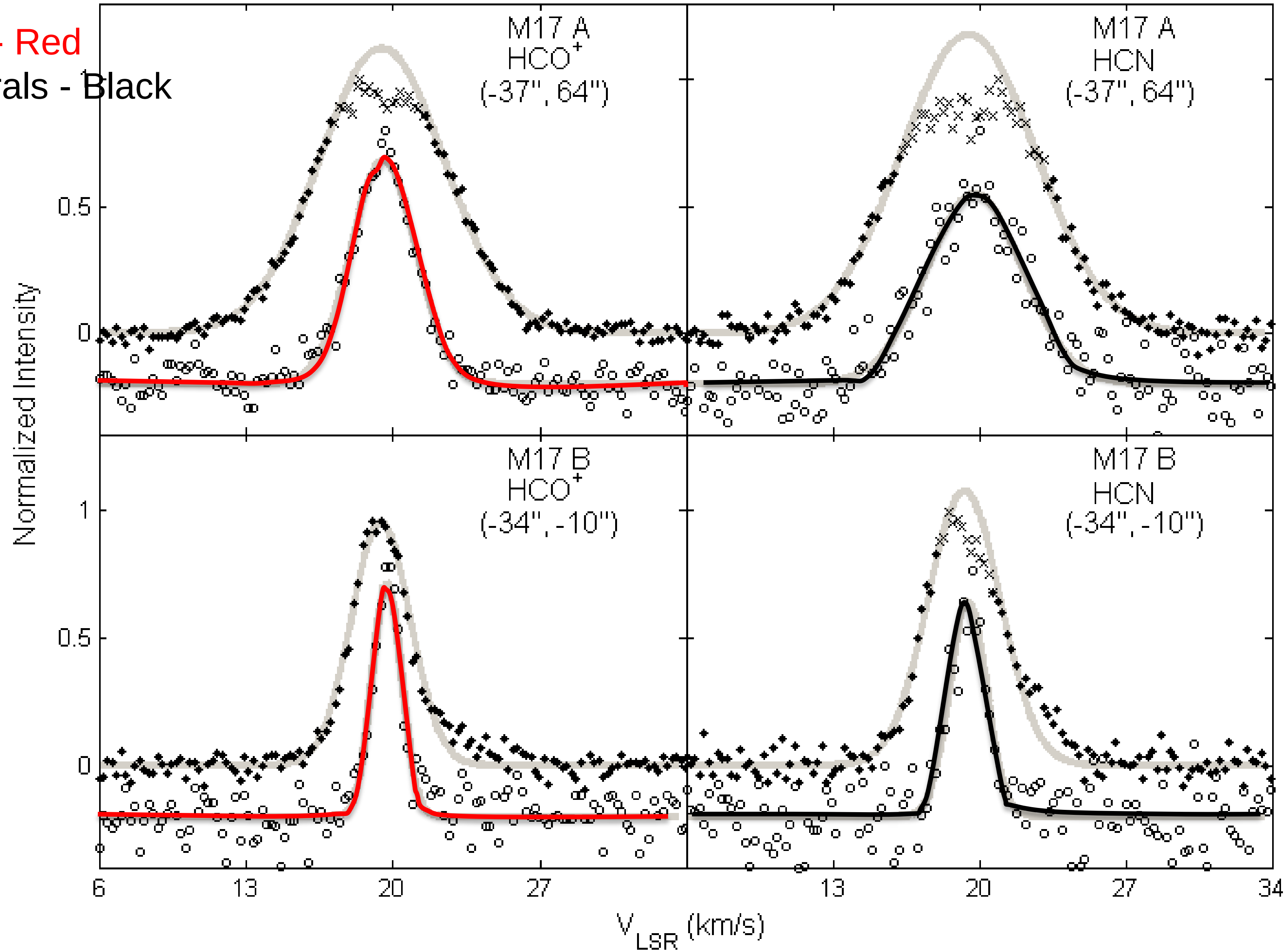
Trends: $\left(L_{AD}\uparrow \text{ as } \rho_i\downarrow\right)$ and $\left(L_{AD}\uparrow \text{ as } \mathbf{B}\uparrow\right)$

(protostellar cores also form on this length scale $\Rightarrow$ it is v. important)

Recall: $\xi \sim 10^{-6}$ to 10^{-8}

In the past: $\mathrm{V}_{A-ion} = B/\sqrt{4\pi\rho_i}$ was deemed too large for practical computations -- The heavy ion approximation (HIA) was the compromise. HIA was found to discard essential physics -- HIA not used here.

Ions - Red
Neutrals - Black

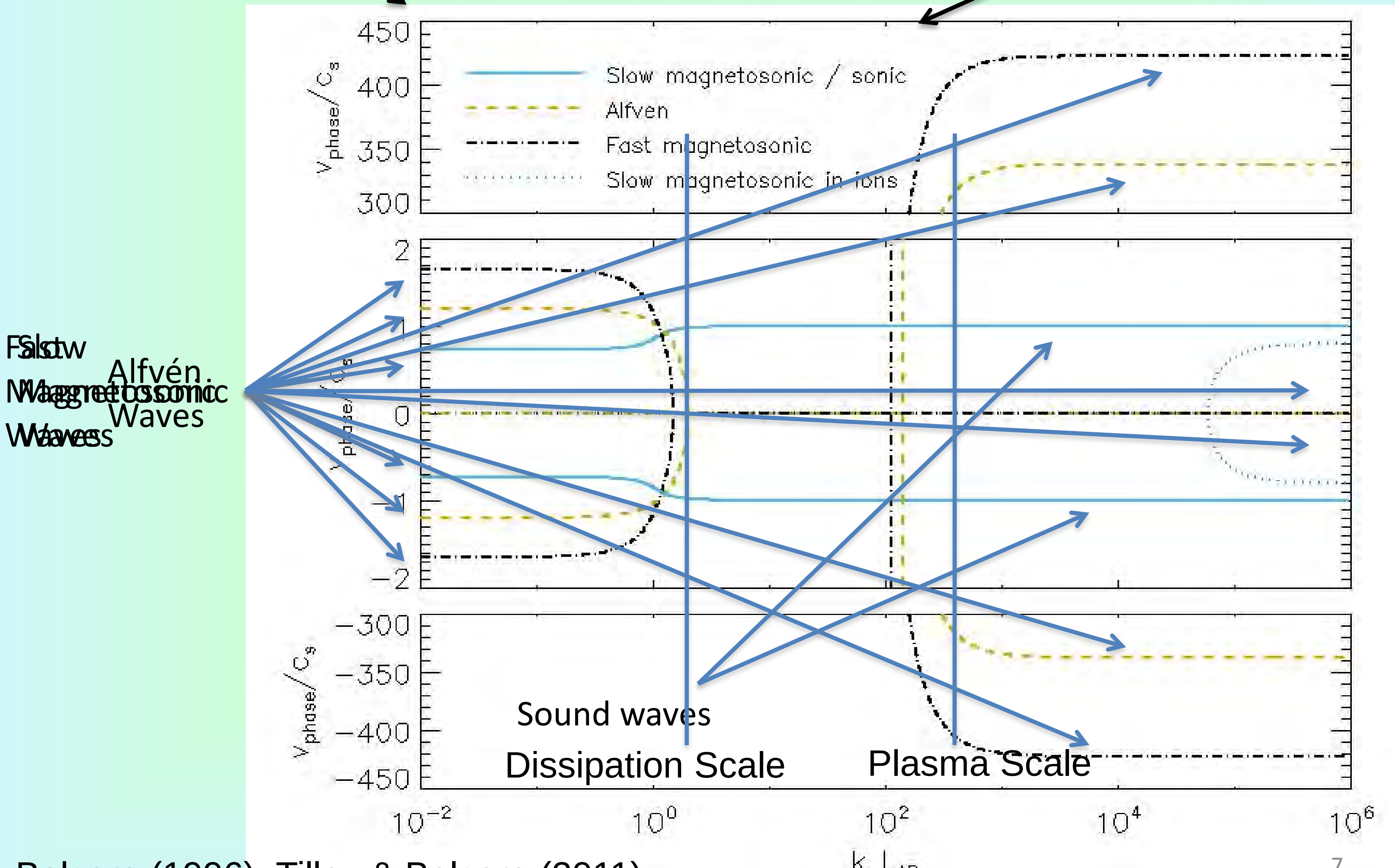


Li & Houde 2008, Li et al 2010 ;
several systems : M17, DR21(OH), Cygnus X, NGC2024

II)Wave Propagation in Partially-Ionized Systems

Large length scales >> L_{AD}

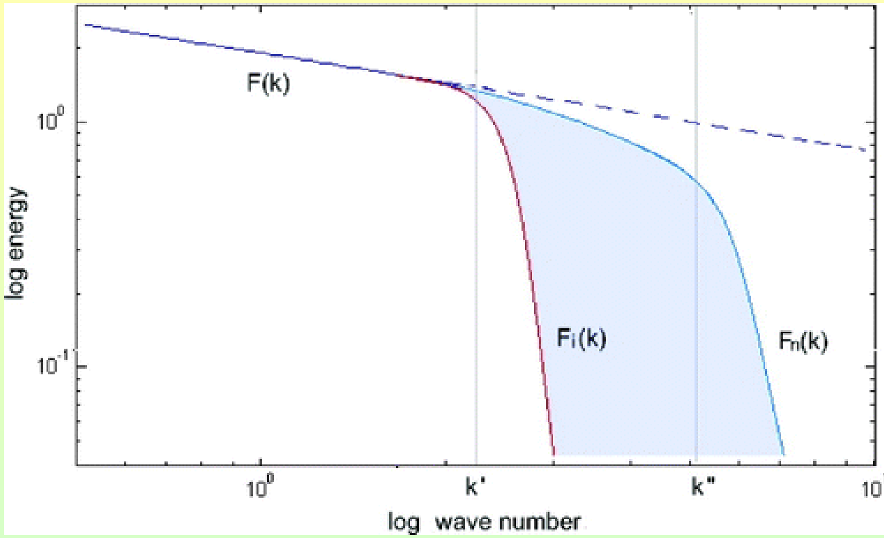
Small length scales << L_{AD}

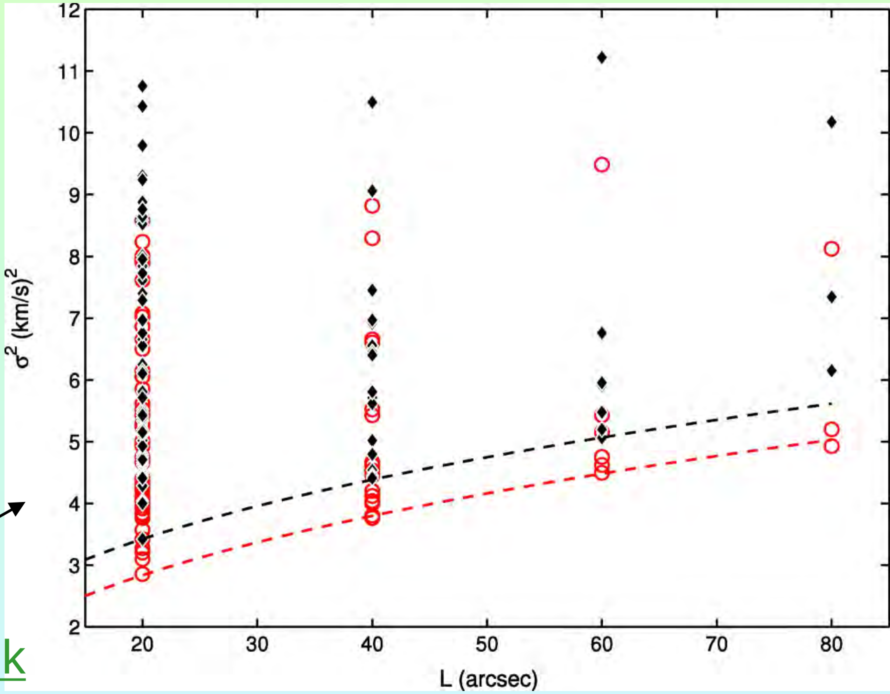


Balsara (1996), Tilley & Balsara (2011)

Understanding the *Small Scale* Turbulence Results

- Ambipolar diffusion sets cutoff length for turbulence in ions; but not for neutrals – Balsara(1996)
- Neutrals dissipate their energy on viscous scale – 5 orders smaller.
- Ions should have attenuated specta or steeper spectral slope than neutrals at (the small) ambipolar diffusion scales – Li & Houde (2008)





Linewidth-size relation for neutrals & ions
Black(HCN) – neutrals ; Red(HCO^+) – ions
This Black and Red is consistent through the talk

III) Simulations of Two-Fluid Turbulence

- RIEMANN code, Balsara 1998, Balsara & Spicer 1999, Balsara 2001b, 2004, Tilley & Balsara 2008, Balsara et al (2009,2011, 2013), Balsara (2012), Balsara & Dumbser (2015), Balsara and Nkonga (2017)
- Same size computational domain & driving.
- Compare ionization fractions from 10^{-2} to 10^{-6}
- Continually driven by adding a spectrum of kinetic energy at large wavelengths
- Alfven speed in ions needs to be resolved – makes time steps v. small & simulations v. challenging

Big Questions:

1) Is there a difference in the character of the turbulence at and beneath the dissipation scale (L_{AD})?

2) How does it reflect on measurements that are made on length scales that are comparable to L_{AD}?

3) What can we learn about the structure and orientation of the magnetic field? Specifically, gradients and relating them to observables.

Driving scale 768 to 384 Δx – Space for (v. small) inertial range to form.

Ambipolar diffusion scale $L_{AD} \sim 120\ \Delta x$

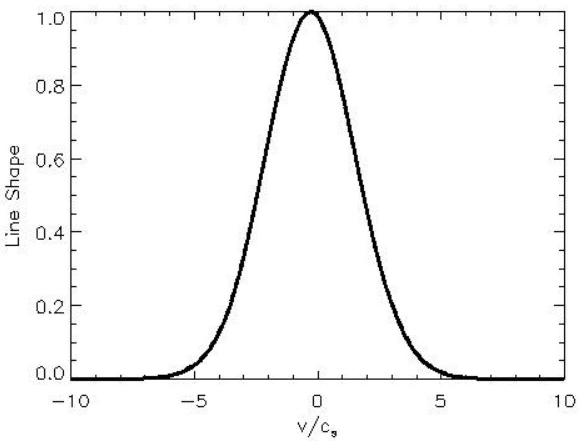
Numerical dissipation dominates on 20 Δx -- Clear separation between ambipolar diffusion and numerical dissipation

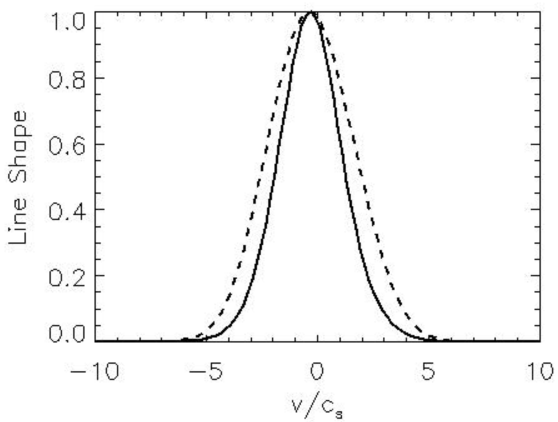
Comparison of Simulated Line Profiles at Different Ionizations

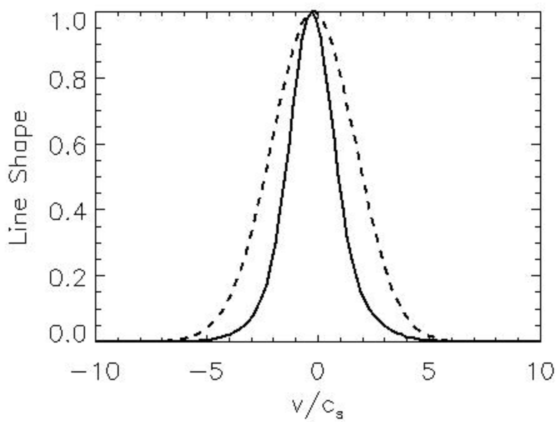
V.V. Imp: Difference between ion and neutral linewidths increases at smaller ionization fractions

Dashed lines – neutrals ; Solid lines – ions

$\xi=10^{-2}$ $\xi=10^{-4}$ $\xi=10^{-5}$



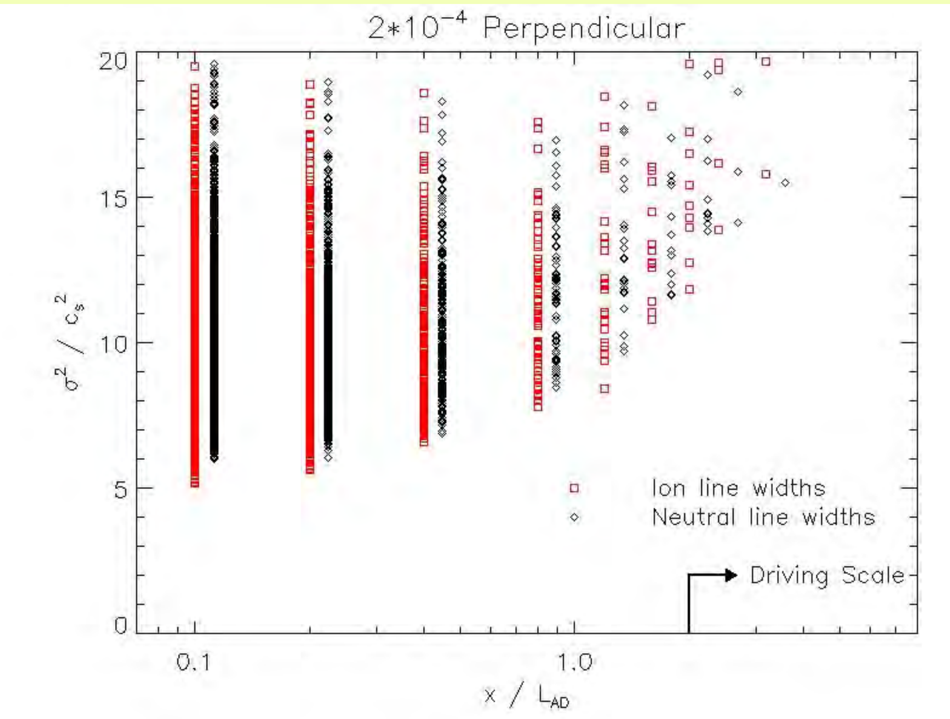




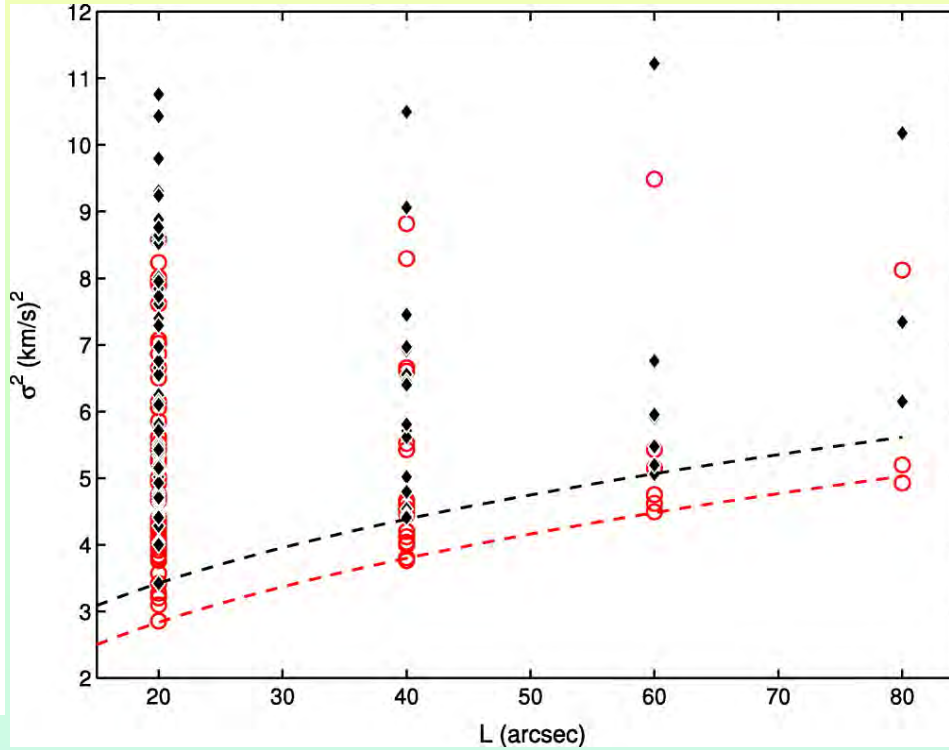
Linewidth-Size Relation – Comparisons with observations!

Built by choosing different sets of zones of different sizes – Similar to observer choosing beams of different angular size.

From Simulations



From Observations



Our simulation results

Ions - Red
Neutrals - Black

Measured line widths (Li & Houde 2008)
(Note that the dissipation scale is at ~0.01 pc, or about 1 arcsec in this figure

IV) Simulations of Two-Fluid Dynamo

Kinematic theory for single fluid dynamo (Batchelor 1950, Kazantsev 1968) not valid for partially ionized gases. Predicts exponential growth of magnetic energy in kinematic regime.

However, most astrophysical plasmas are partially ionized.

New kinematic theory developed by Xu and Lazarian (2016) for dynamo growth in partially ionized plasmas. Predicts quadratic growth of magnetic energy in kinematic regime.

Physical reason: Magnetic energy is always below equipartition and, therefore, experiences strong damping. Diffusion is from two-fluid ambipolar plasma effects.

Non-linear regime remains the same because turbulent diffusion dominates physical diffusion. Predicts (slower) linear growth of magnetic energy.

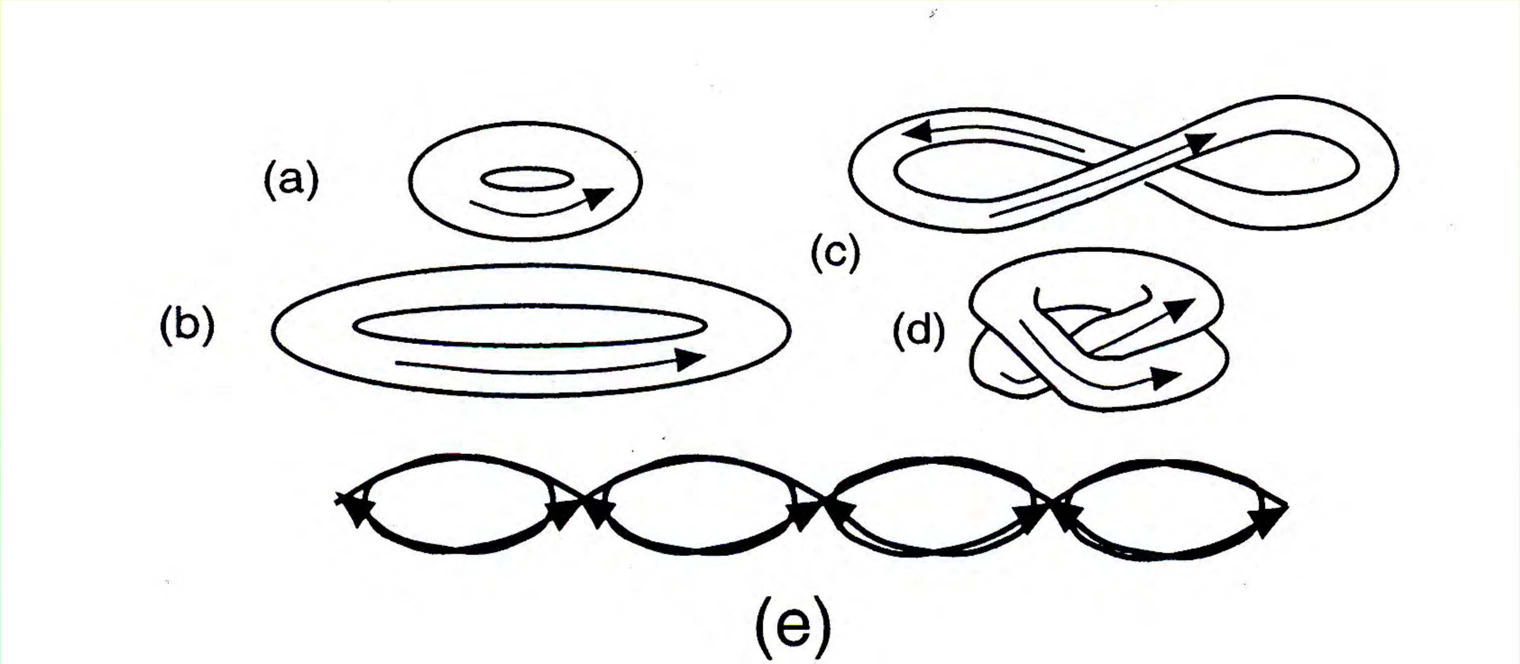
High resolution Numerical Simulations (1024^3 zones and up) can tell the difference between exponential and quadratic growth. Can also verify theory in non-linear regime.

Applications to:

Molecular Clouds	Magnetic field amplification	Cosmic Ray Scattering
Supernova Remnants	Gradients for measuring magnetic fields	

Theoretical Background : Fast or Small-scale Dynamos

1) By a sequence of Stretch, Twist and Fold operations we can grow **B**. Known as the STF dynamo. Note: These are vigorous motions that scramble the mean field!



2) STF dynamo is kinematical. Small-scale dynamo theories that include dynamics have also been constructed.

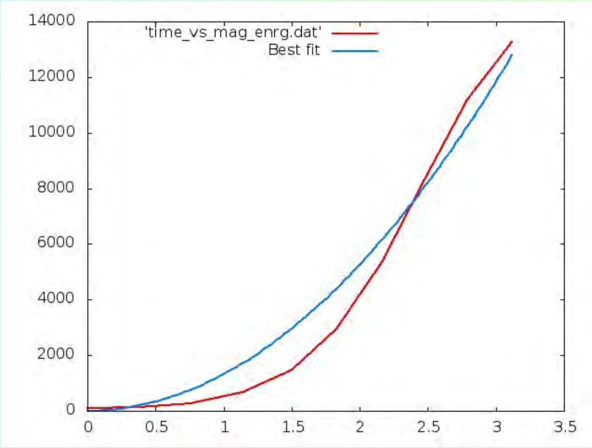
3) There is a competition between STF which causes magnetic field growth and turbulent diffusion, which causes dissipation of magnetic field.

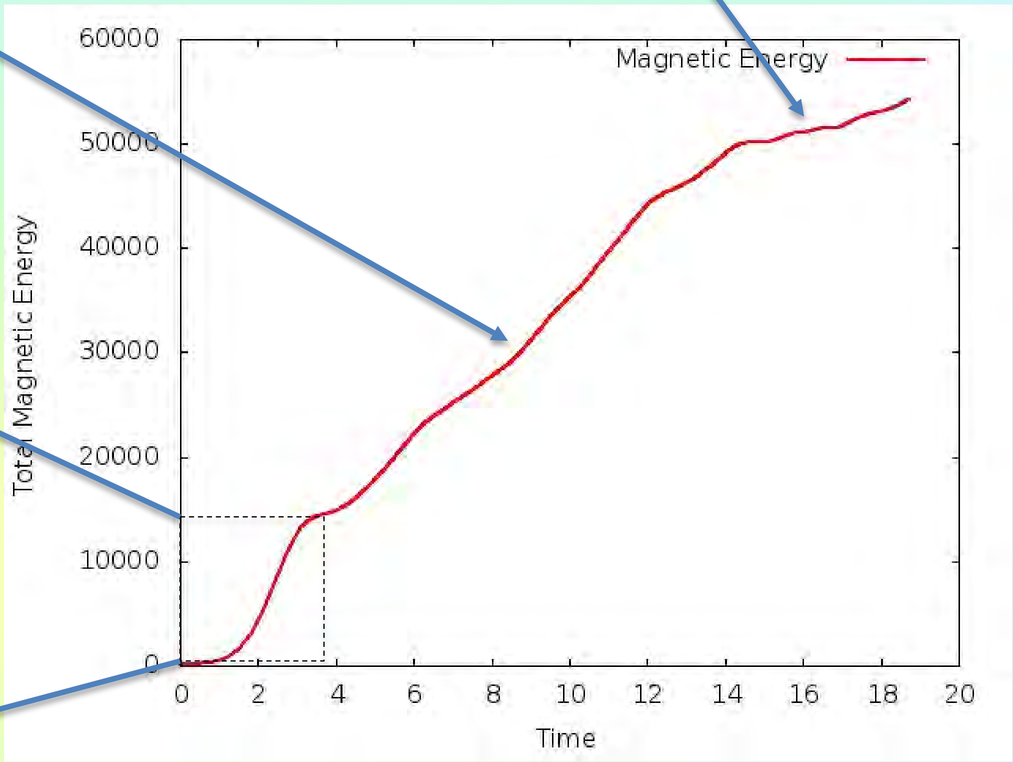
4) Both the above predict growth times that can be relatively fast; as eddy turnover times.

Non-Linear Growth

Non-Linear Saturation

Kinematic Regime;
best fit to quadratic



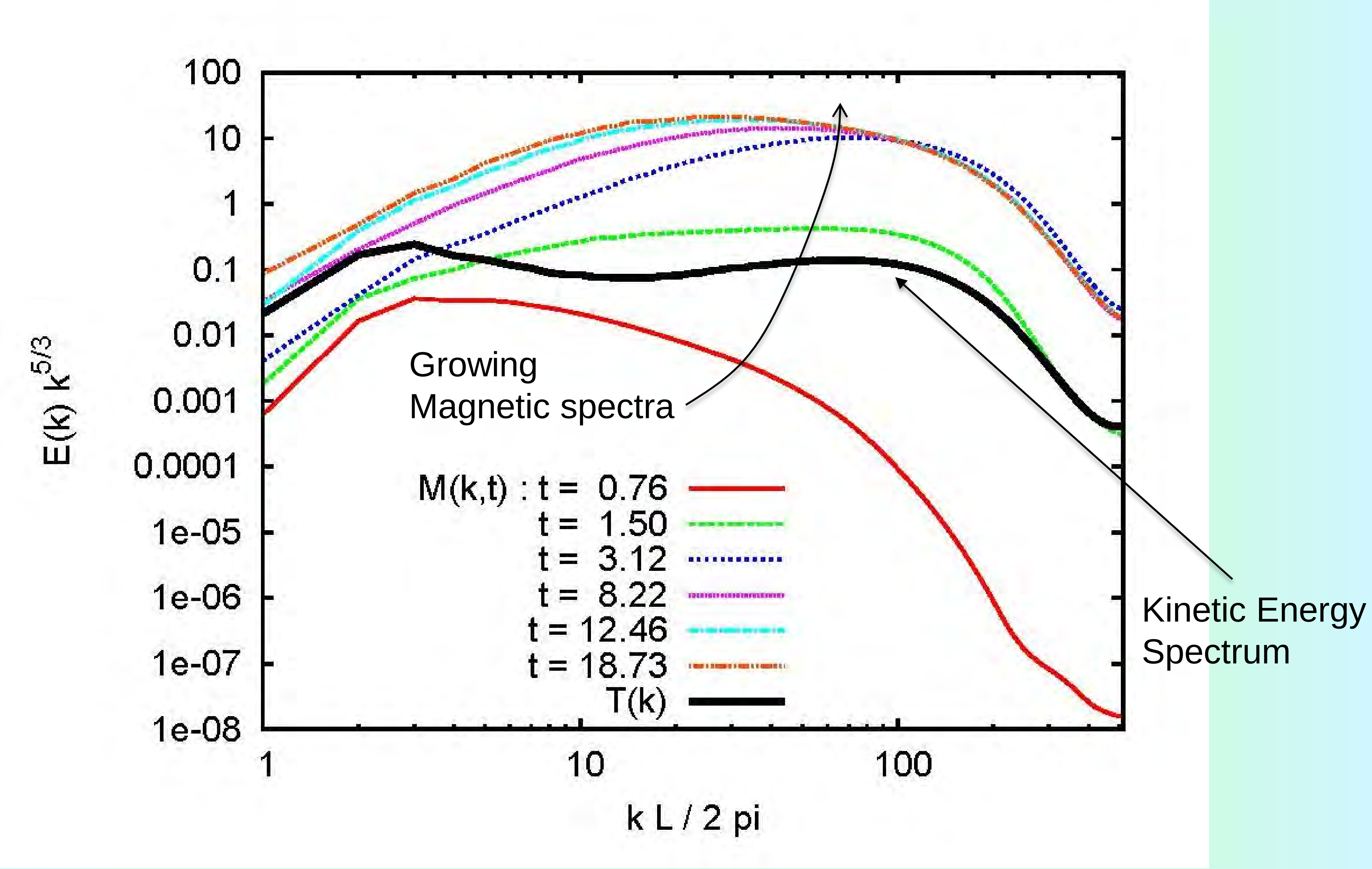


Magnetic energy initially grows quadratically with time in the kinematic regime.

As magnetic energy equipartitions with kinetic energy on a given scale, the magnetic energy grows linearly in the non-linear regime.

Non-linear saturation also verified.

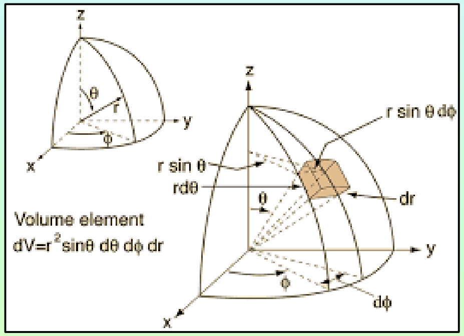
Compensated Kinetic and Magnetic Energy Spectra

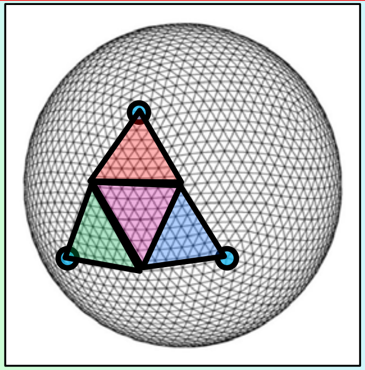


Simulations Verify Kazantsev theory and the hypothesis of Xu and Lazarian

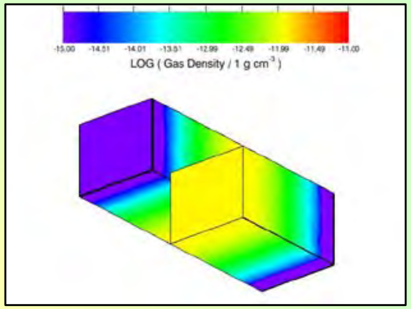
V) Geodesic Mesh MHD: A New Paradigm for Computational Astro.

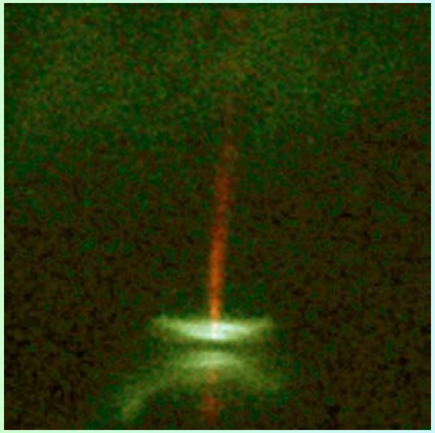
“On Being Round”





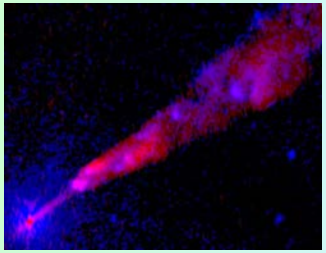
Problem: Several Astrophysical systems are spherical;
Codes for simulating them have been logically Cartesian. (r-θ-ϕ coordinates)



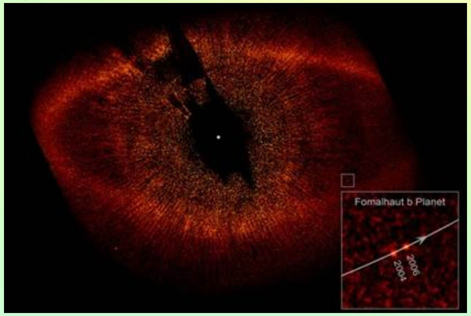


Example systems:-
Accretion Disks and MRI – Done in Shearing Sheet boxes

Jets propagating in pressure gradients around Galaxies



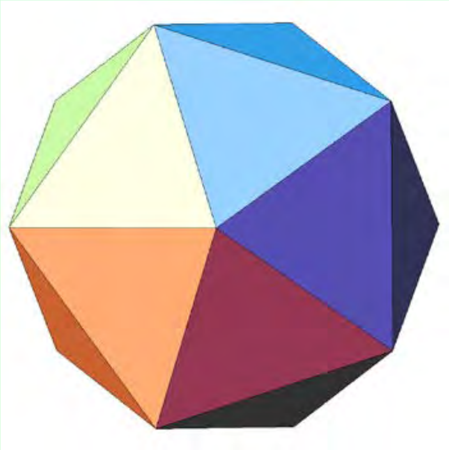
Star and Planet Formation



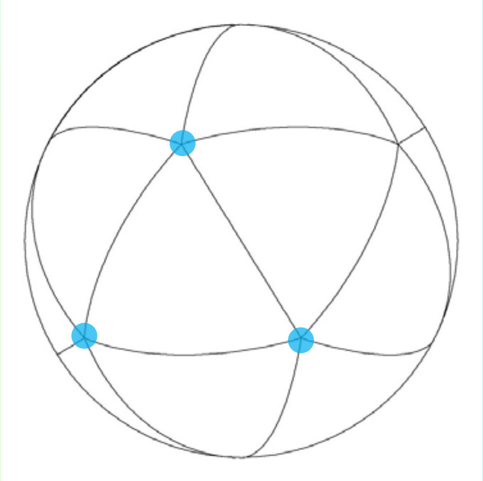
Other Applications:-
Heliosphere Magnetospheres of planets Convection in the Sun
Convection in AGB Stars Supernovae
Possible uses in Galaxy formation Possible uses in NS-NS collisions

Tesselation of the Sphere (Sectorial Subdivision)

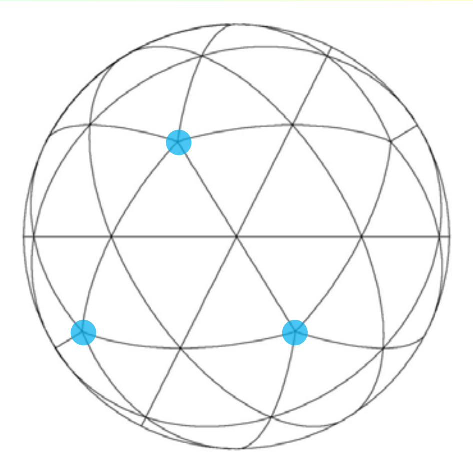
Icosahedron



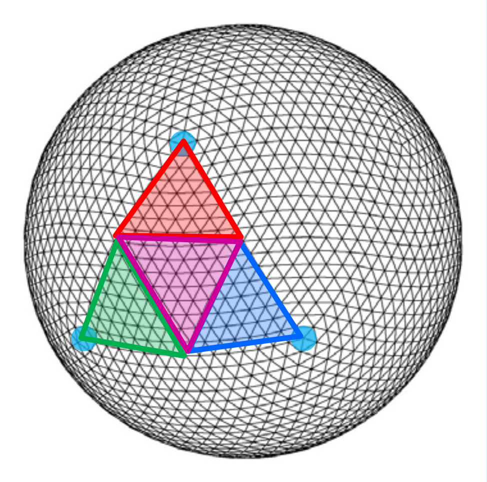
Spherical Icosahedron



Level 0 geodesic mesh – 20 great spherical triangles bounded by great circles. Each triangle is called a sector so we have 20 sectors. Each sector makes an angle of 45^o w.r.t. the center.



Level 1 sector division – we have 80 sectors. Each sector makes an angle of 22.5^o w.r.t. center.



Level 4 zoning within each level 1 sector. Each zone makes an angle of 2.8125^o w.r.t. center.

- Opportunities for efficient processing.
- Opportunities for parallelism.

High Accuracy Divergence-Free MHD on Geodesic Meshes – Algorithmic Issues

Built on the following four easy steps:-

i) High order WENO Reconstruction on Unstructured Meshes.

ii) Divergence-free reconstruction of magnetic fields.

iii) Genuinely Multidimensional Riemann Solver.

iv) High Order Temporal Update. Runge-Kutta or ADER at hi order.

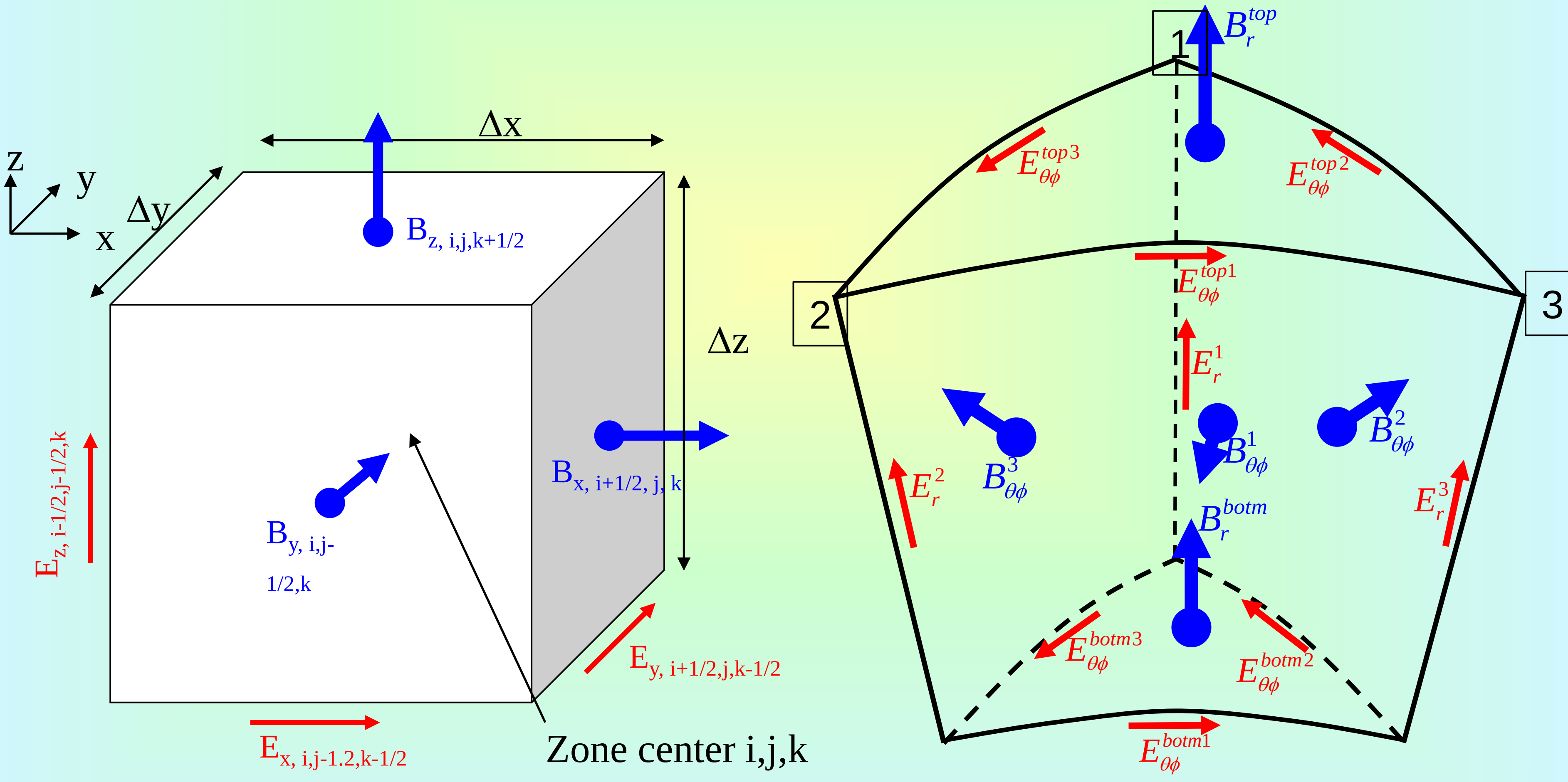
We have made all higher order extensions. Results shown in next section.

This need for **higher order accuracy** is motivated by the fact that astrophysicists are beginning to face up to the presence of **MHD turbulence**. Such problems have **strong shocks**; we must handle shocks.

Turbulence simulations always require the **lowest possible numerical dissipation and dispersion**. High order accuracy is the only known way of beating down dissipation and dispersion.

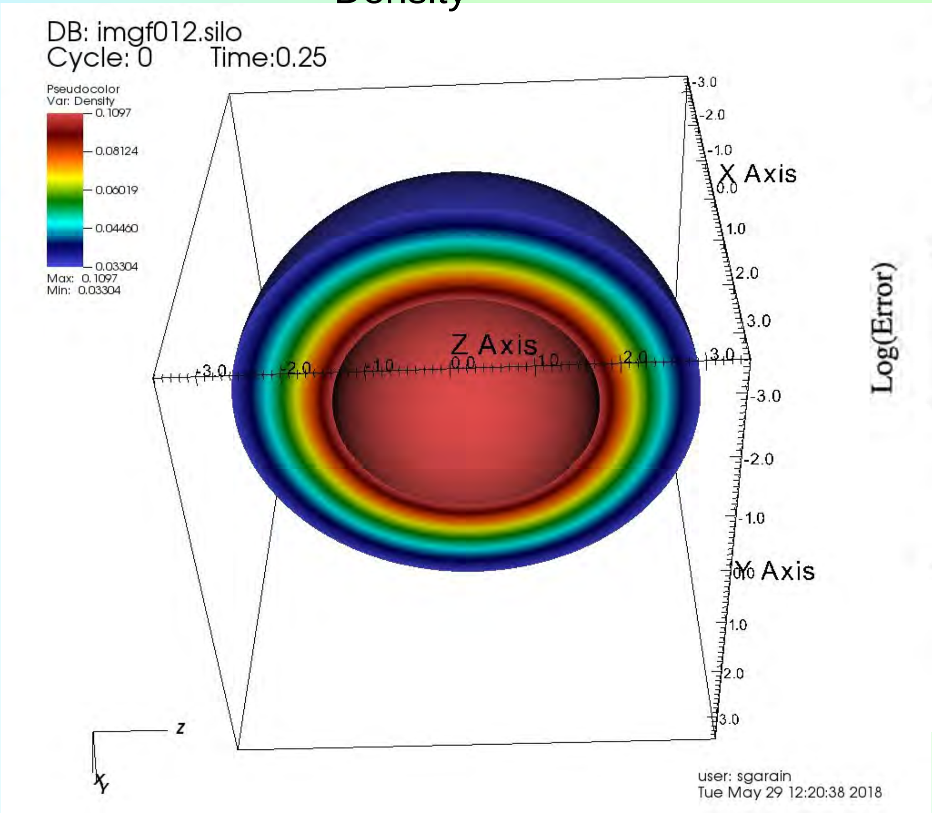
We accurately preserve the analogy between the divergence-free update on rectangular meshes with the update on frustrums!

This Goal is exactly provided to us by the Multidimensional Riemann Solver.

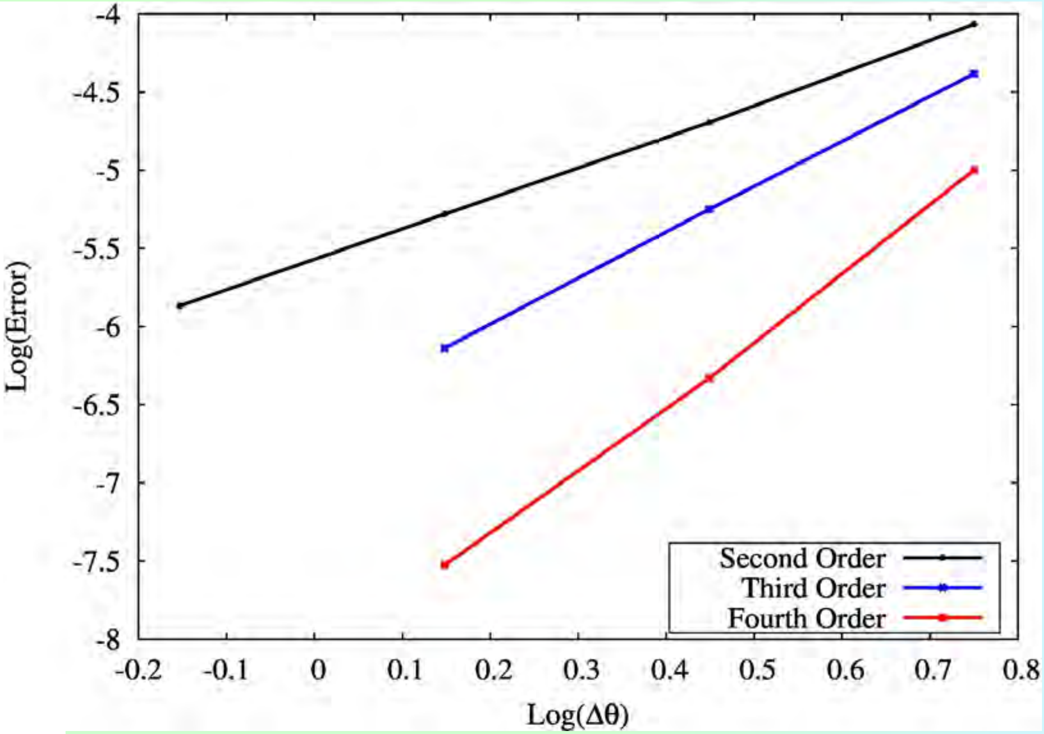


Results: Solar Wind

Density

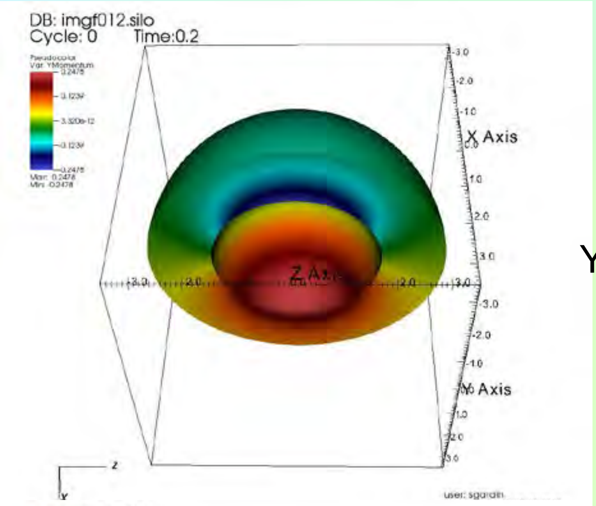


Accuracy

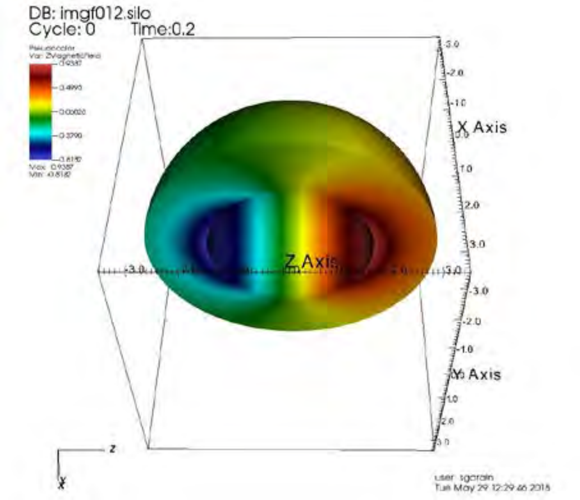


$\Delta\theta$ is measured in degrees.

Results: MHD Outflow with Method of Manufactured Solution Wind

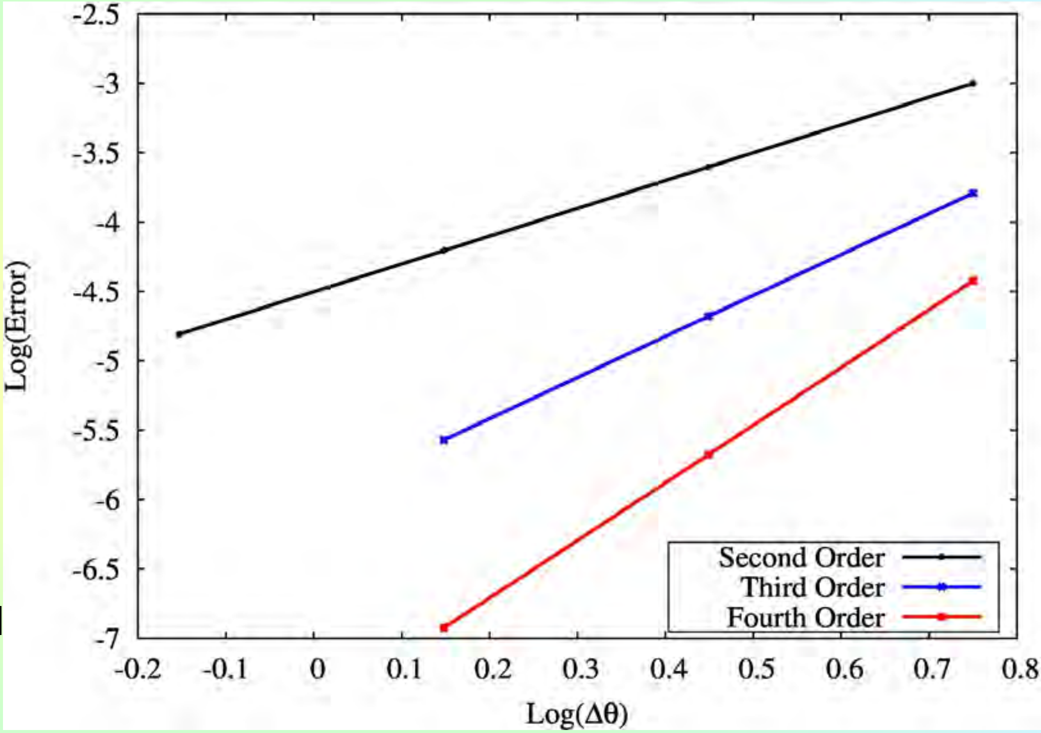


Y-Momentum



Z-Magnetic Fiel

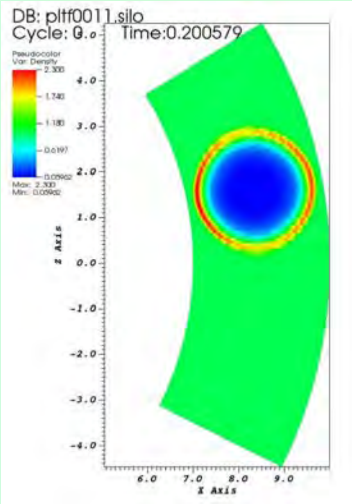
Accuracy



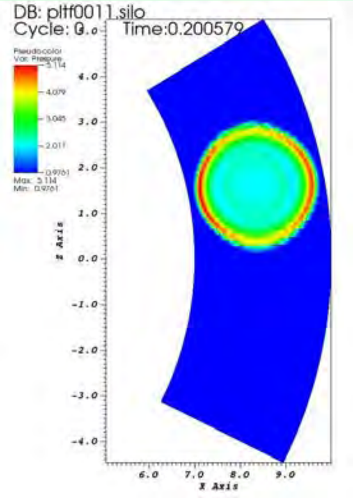
$\Delta\theta$ is measured in degrees.

Results: MHD Blast Problem on Spherical Geodesic Mesh

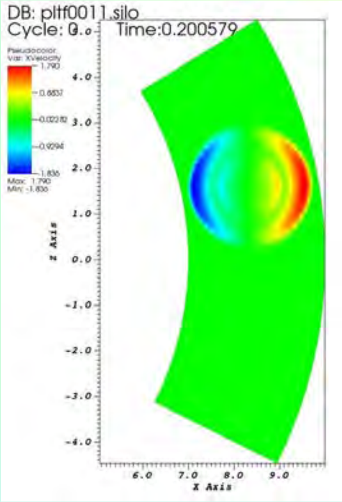
Density:



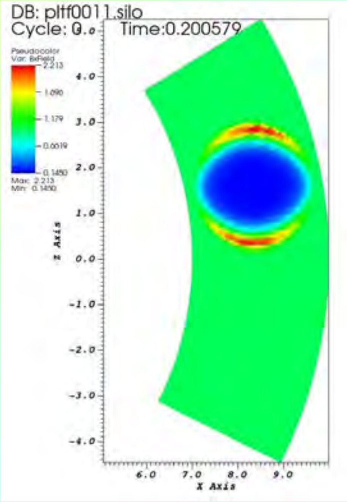
Pressure:



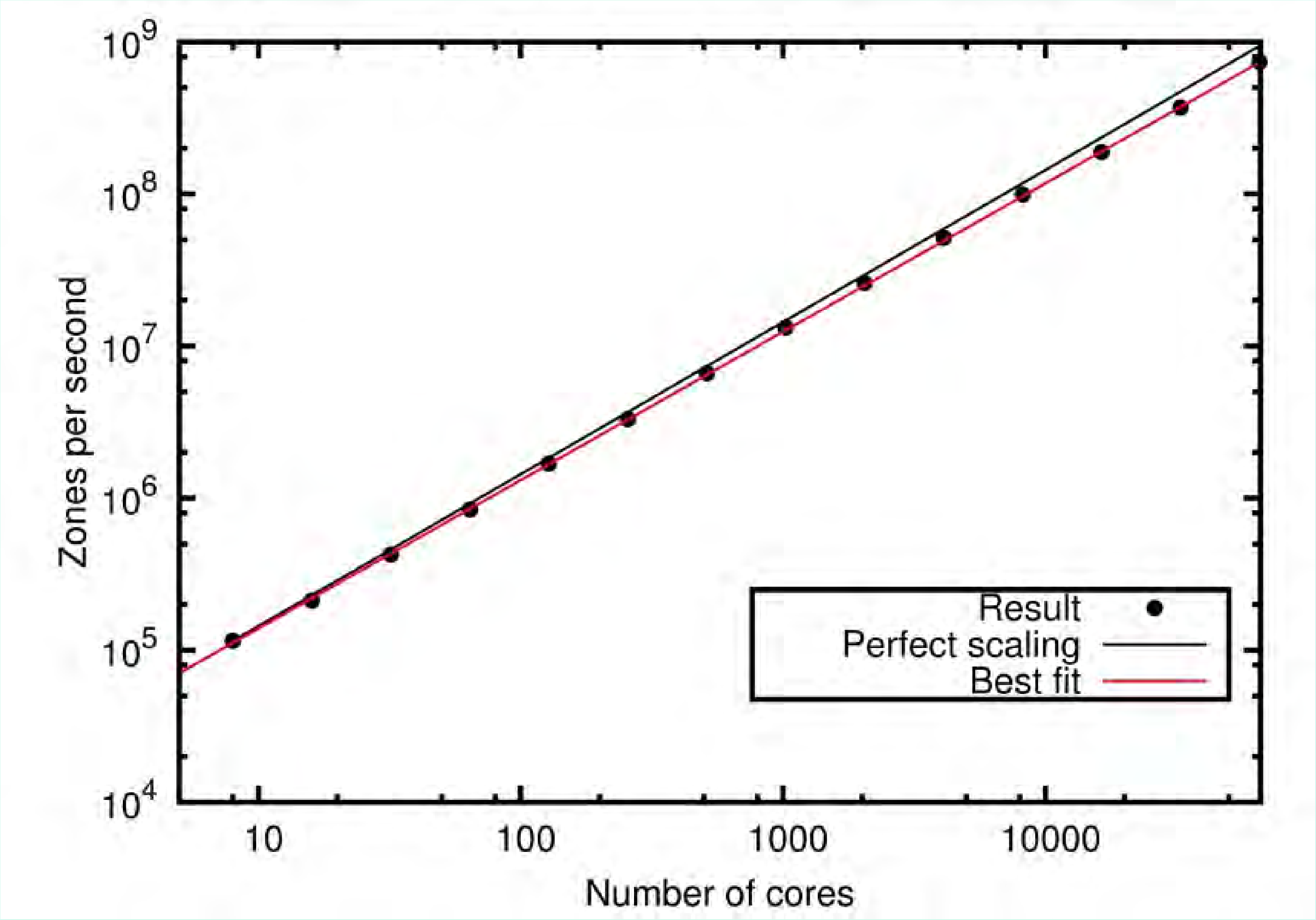
X-Velocity:



X-Magnetic Field:



Results: Exceptional Scalability of Geomesh MHD Code on Blue Waters



VI) Conclusions

1) Study of wave propagation provides important insights.

2) Line profiles and Linewidth-size relationship shows a prominent difference between ions and neutrals. This is especially so when l.o.s. is perpendicular to the magnetic field.

3) Dynamo Simulations match precisely with the theory.

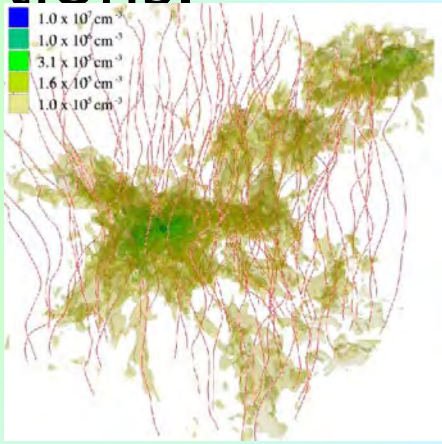
4) A new paradigm for treating Geodesic Mesh MHD has been invented. It is a game-changer for Computational Astrophysics and it scales exceptionally well on Blue Waters.

5) Our work (via improvements to GNU compiler) has led to infrastructural improvements on all PetaScale and XSEDE resources.

Turbulence-Driven Star Formation

- Focus on the pc scales where supersonic turbulence is observed – cores form where streams collide
- Cloud dynamics driven by internal stirring, not magnetic fields. SNR-driving; Winds; Jets
- Model requires magnetic pressure << gas pressure to form cores. Inconsistent with observations.

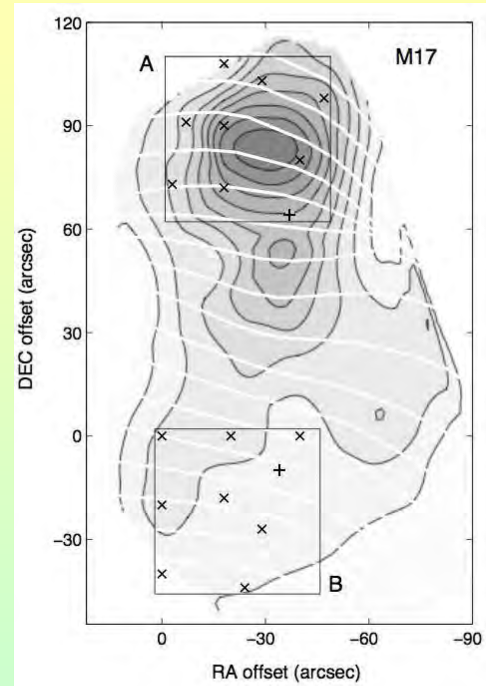
  Tilley & Pudritz 2007

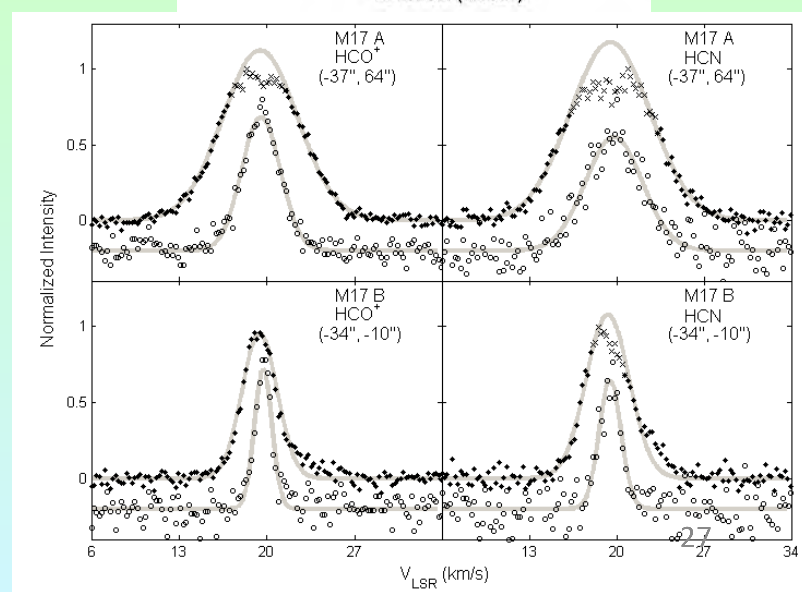


- Key Challenges:
  - Evolution occurs very quickly (10^5 years)
  - How do molecular clouds survive over 10^7 years?
  - How do prestellar cores (which are observed to collapse subsonically, i.e. not on dynamical times) regulate their collapse?

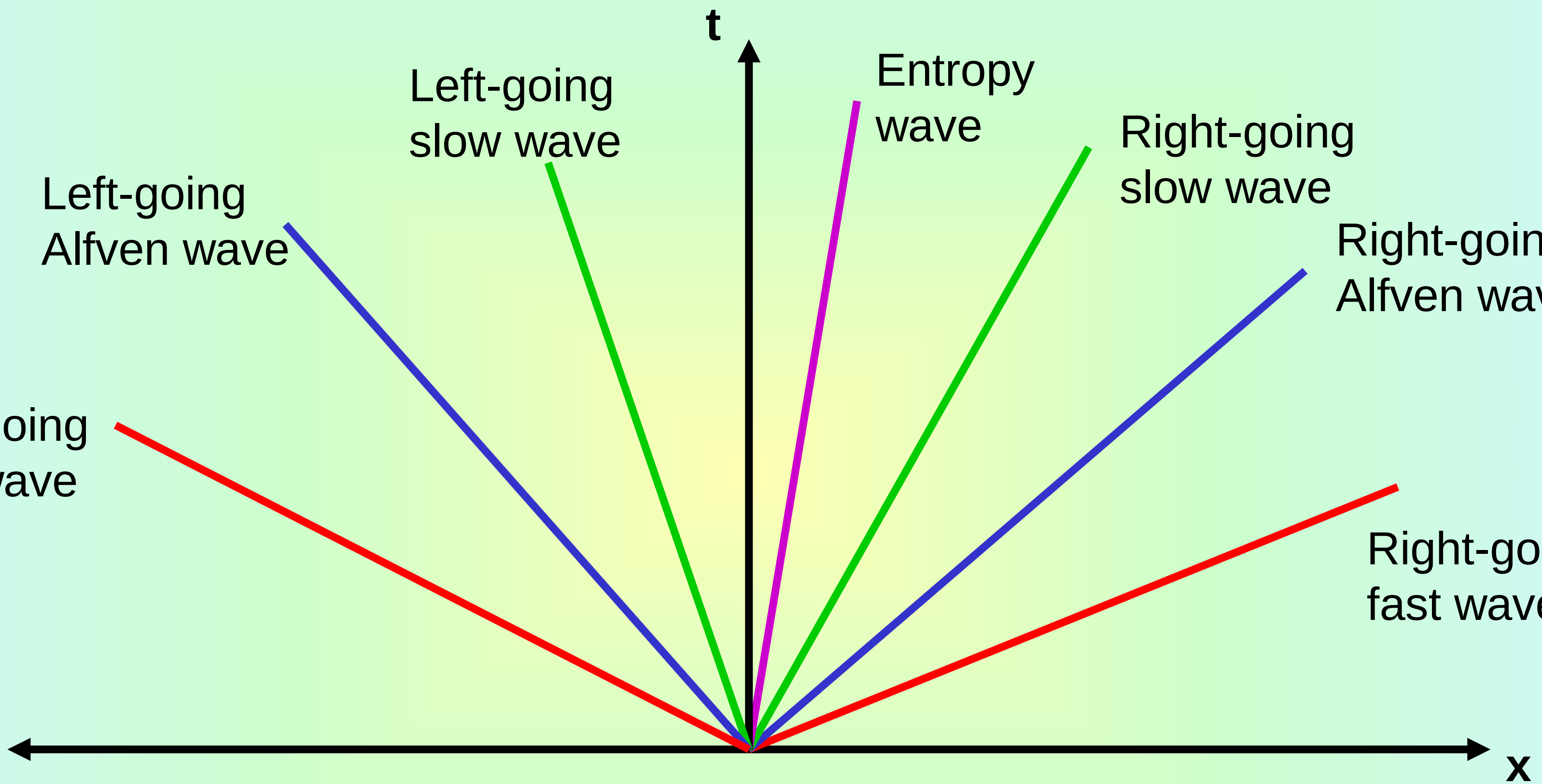
Properties of the Turbulence (*on smaller scales*)

- Star formation takes place on smaller scales; thus important.
- Turbulence is expected to form an energy cascade – numerous observations on large scale.
- Li & Houde (2008) observed that the turbulent velocity of **ions (HCO^+)** was smaller than that of **neutral (HCN)** molecules
  - difference in the turbulence spectrum?
- Li et al. (2010): M17, DR21(OH), NGC 2024 show similar trends





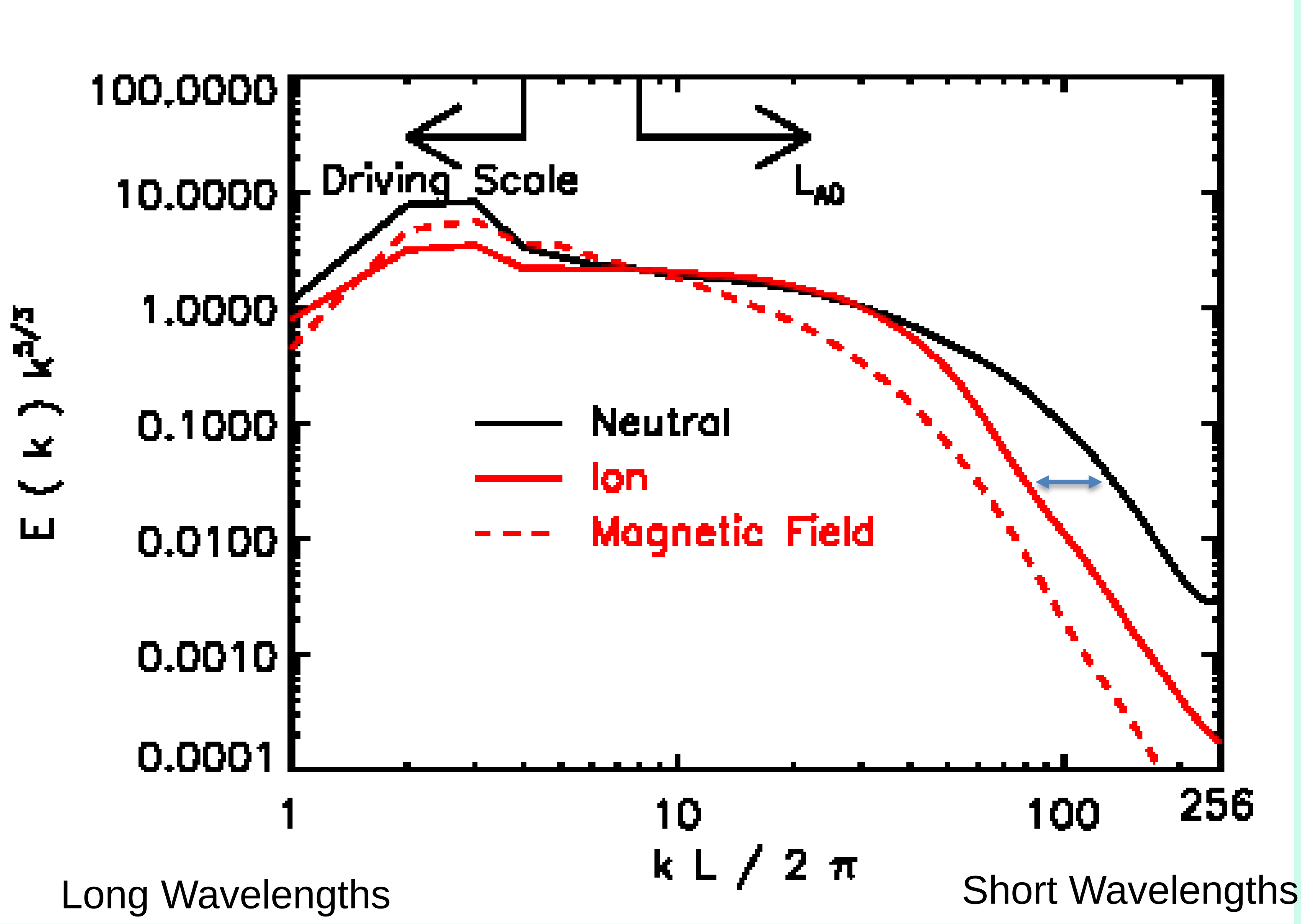
II)Wave Propagation in Partially-Ionized Systems



Space-Time Diagram for ***IDEAL*** MHD Waves

On length scales >> L_{AD}, two-fluid MHD and ideal MHD are identical

Compensated Energy Spectrum Run A3 – Weakly-Ionized/Weakly Coupled (10^{-4}) Recall that $L_{AD} \uparrow$ as $\rho_i \downarrow$

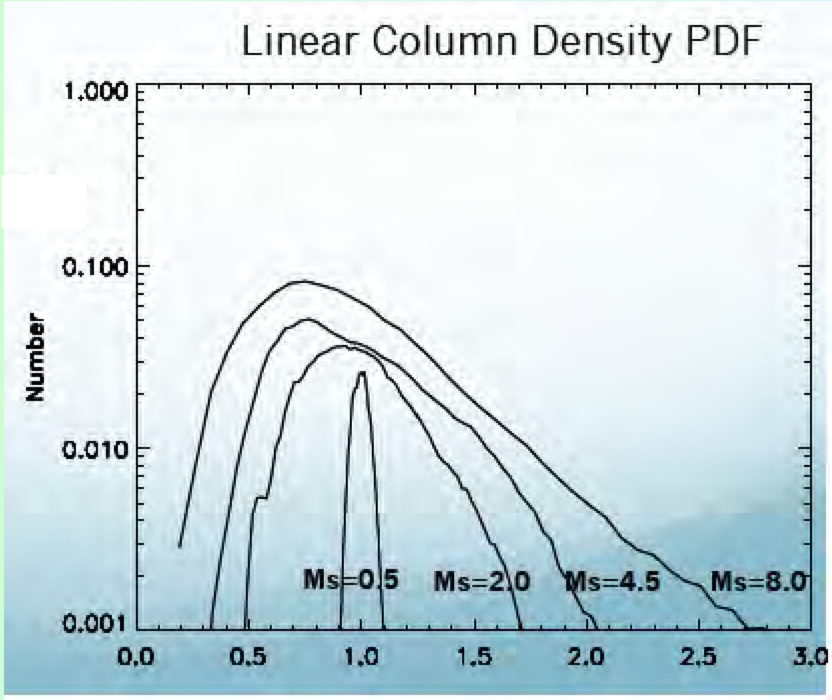


Notice the separation between ions and neutrals velocity spectrum.
Small inertial range can also be seen. Spectrum more shock-like ~ k^{-2}

Switching Gears: Density PDFs from Single Fluid MHD Turbulence Simulations

These are easy to obtain for observers and easy to extract from simulations

- Show anisotropy due to the magnetic field
- Correlation lengths dependent on magnetic field strength, sonic Mach number
- Statistical moments as a function of sonic, Alfvenic Mach numbers
- Observational diagnostics to help identify magnetic field directions, strengths.
- Skewness and kurtosis can be extracted.

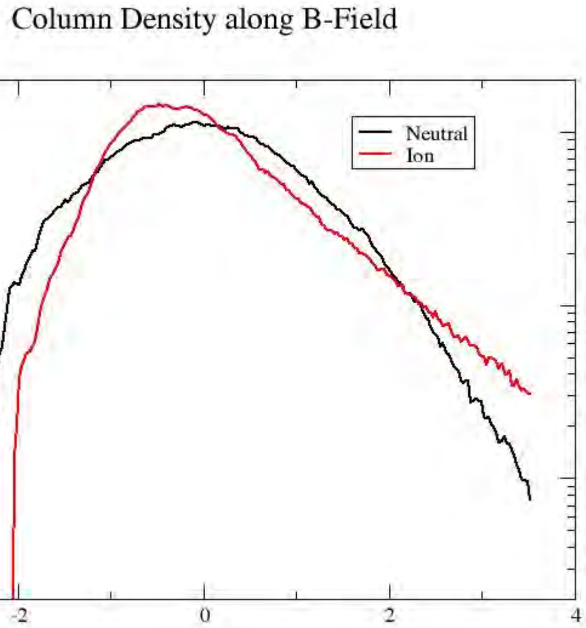


Burkhart+(2011)
Kowal, Lazarian Beresnyak(2007)

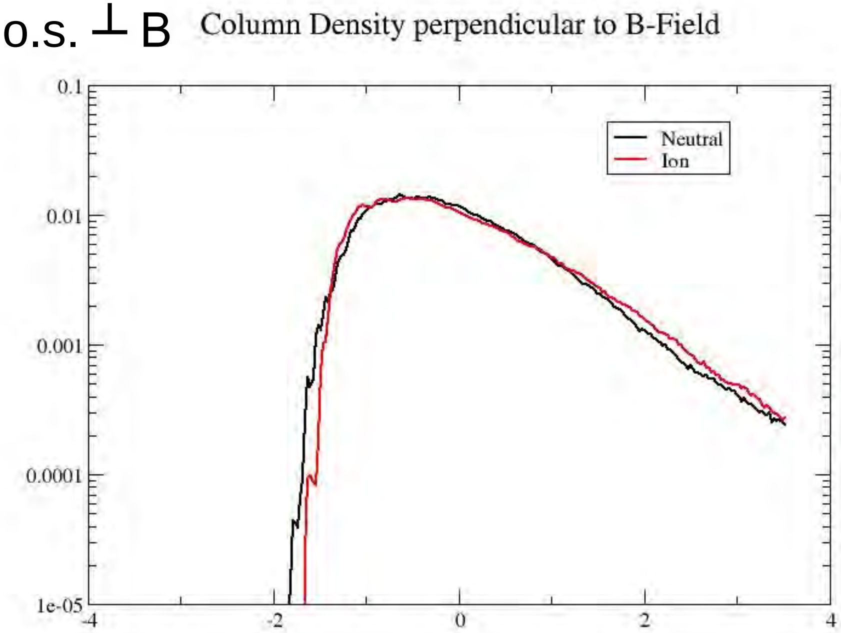
Density PDFs from Two-Fluid MHD Turbulence Simulations

Run A6 Alfven speed = 6 c_s

l.o.s. || B



l.o.s. ⊥ B



V.V.Imp: Maximum Difference in density PDFs **seen along B-field; none perpendicular to it.**

This trend anti-correlates with the linewidth-size relationship ➔ New observational diagnostic on the 3D structure of the B-field!

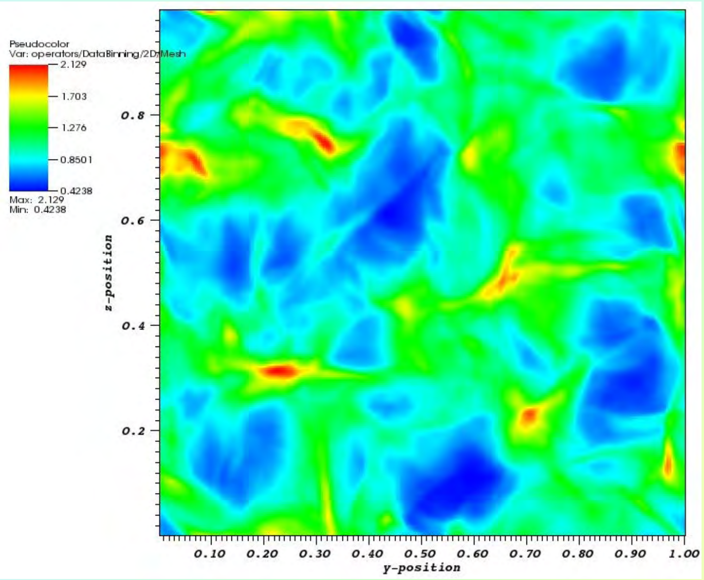
Stronger fields show more of this trend!

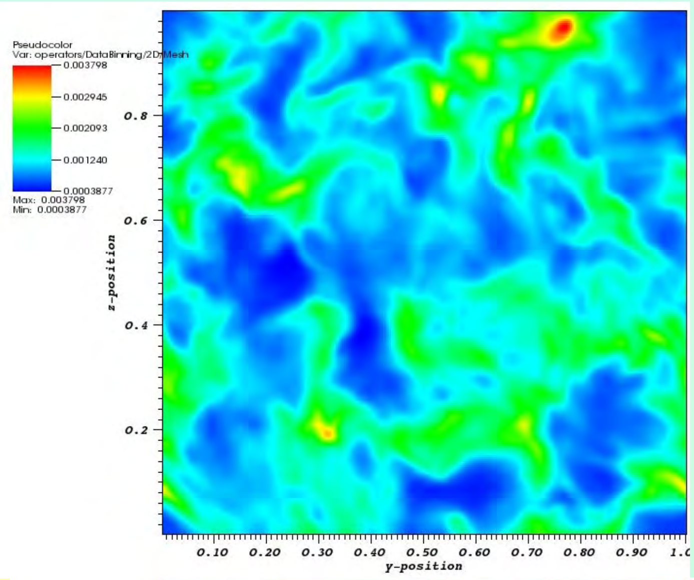
Skewness and kurtosis can also be extracted from such data. Prominent differences when l.o.s. is along B.

Simulated Density maps

Neutrals **Ions**

l.o.s II B





l.o.s. ⊥ B