Policy Responses to Climate Change in a Dynamic Stochastic Economy

Kenneth Judd[1]
Yongyang Cai[2]

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[1]Paul H. Bauer Senior Fellow, Hoover Institution (Stanford, CA); Presenter and Senior representative for Blue Waters GLCPC project (Lars Hansen PI).
[2]Senior Research Scientist, University of Chicago.

Introduction

Economics is a complex system.
Economics research ignores this

- Economists analyze simple stylized models of pieces of the system
- Pencil and paper preferred to computers and code

We are trying to change that

- Create robust and general tools that can use state-of-the art numerical methods on modern computer architectures
- Climate change policy is the application

Climate Change Policy Analysis

Question: What can and should be the response to rising CO_2 concentrations?

- Analytical tools in the literature: IAMs (Integrated Assessment Models)
  - Two components: economic model and climate model
  - Interactions: Economy emits CO_2 that raises world average temperature that reduces economic productivity.
- Existing IAMs cannot study dynamic decision-making in an evolving and uncertain world
  - Most are deterministic where economic actors know perfectly future economic and climate events.
  - Limitations are due to economists' aversion to modern computational tools

Uncertainty and Risk

All agree that uncertainty needs to be a central part of any IAM analysis
Multiple forms of uncertainty

- Risk: productivity shocks, taste shocks, uncertain technological advances, weather shocks
- Parameter uncertainty: policymakers do not know parameters that characterize the economic and/or climate systems
- Model uncertainty: policymakers do not know the proper model or the stochastic processes

Theme of our work

- We can pursue quantitative studies with far fewer simplifications
- We can incorporate modern models of macroeconomic systems
- We can pursue uncertainty quantification (UQ)

Cai-Judd-Lontzek DSICE Model

Extends Nordhaus' DICE model

- Climate system
  - Carbon mass: $\mathbf{M}_t = (M_t^{\mathrm{AT}}, M_t^{\mathrm{UP}}, M_t^{\mathrm{LO}})^\top$
  - Temperature: $\mathbf{T}_t = (T_t^{\mathrm{AT}}, T_t^{\mathrm{LO}})^\top$
  - Carbon emission: $E_t = \sigma_t(1-\mu_t)Y_t + E_t^{\mathrm{Land}}$
  - Radiative forcing: $F_t = \eta \log_2 \left(M_t^{\mathrm{AT}}/M_0^{\mathrm{AT}}\right) + F_t^{\mathrm{EX}}$
- Economic system:
  - gross output: $Y_t \equiv f(k_t, \zeta_t, t) = \zeta_t A_t k_t^\alpha L_t^{1-\alpha}$
    - productivity state $\zeta_{t+1} = g^\zeta(\zeta_t, \chi_t, \omega_t^\zeta)$ is stochastic productivity process
    - the long run risk process, χ_t, is very persistent
  - damage factor: $\Omega_t \equiv \left(1 + \pi_1 T_t^{\mathrm{AT}} + \pi_2 (T_t^{\mathrm{AT}})^2\right)^{-1}$
  - emission control cost: $\Lambda_t \equiv \psi_t^{1-\theta_2}\theta_{1,t}\mu_t^{\theta_2}$, where μ_t is policy choice
  - output net of damages and emission control: $\Omega_t(1-\Lambda_t)Y_t$

Dynamic Optimization Problem

- Epstein-Zin Preferences: recursive utility function
  - $u(C, L) = \frac{(C_t/L_t)^{1-1/\psi}}{1-1/\psi} L_t$: utility flow per period
  - ψ: dynamic consumption flexibility (default: 1.5)
  - γ: risk aversion (default: 10)
  - $\Gamma = \frac{1-\gamma}{1-1/\psi}$: composite factor for preferences
- State: $\mathbf{x} = (k, \mathbf{M}, \mathbf{T}, \zeta, \chi)$
- Bellman equation ($V_{300}(\mathbf{x})$ is fixed, and is the terminal condition)

$$
\begin{aligned}
V_t(\mathbf{x}) = \max_{c,\mu} \quad & u(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ \left(V_{t+1}\left(\mathbf{x}^+\right)\right)^{\Gamma} \right\}\right]^{1/\Gamma}, \\
\text{s.t.} \quad & k^+ = (1-\delta)k + \Omega_t(1-\Lambda_t)Y_t - C_t, \\
& \mathbf{M}^+ = \Phi^{\mathrm{M}}\mathbf{M} + (E_t, 0, 0)^\top, \\
& \mathbf{T}^+ = \Phi^{\mathrm{T}}\mathbf{T} + (\xi_1 F_t, 0)^\top, \\
& \zeta^+ = g_\zeta(\zeta, \chi, \omega_\zeta), \\
& \chi^+ = g_\chi(\chi, \omega_\chi),
\end{aligned}
$$

General Operators in Economics Models

- Economics problems can be modeled as difference operator equations on Banach spaces
  - A function $V_t(\mathbf{x})$ represents economic system at time t as a function of $\mathbf{x}$
  - Operator equation is $V_t(\mathbf{x}) = \mathfrak{F}_t V_{t+1}(\mathbf{x})$, $t = 0, 1..., T-1$ in appropriate Banach space
  - Terminal condition: $V_T(\mathbf{x})$ known for time T
- Solve backwards in time, like Hamilton-Jacobi-Bellman PDEs
  - People are not particles
  - Decisions today depends on expectations of what will be done tomorrow
- Numerical challenges
  - Approximate $V_t(\mathbf{x})$ functions over a compact domain in Euclidean space - difficult, need to avoid curse of dimensionality
  - Approximate $\mathfrak{F}_t$ operator - easy if you use quadrature
  - Solve optimization problem - easy with good code (NPSOL)

Anisotropic Method for Efficient Approximation

We develop a flexible anisotropic approximation; it is adaptive in that we check accuracy at each iteration

- Anisotropic approximation nodes: $N = \prod_{i=1}^{d} m_i$
- Anisotropic Chebyshev polynomial approximation
  - notation: $\phi_\alpha(\mathbf{x})$ is product $\phi_{\alpha_1}(\mathbf{x})\phi_{\alpha_2}(\mathbf{x})..\phi_{\alpha_d}(\mathbf{x})$
  - degrees $(n_1, ..., n_d)$

$$\hat{V}(\mathbf{x};\mathbf{b}) = \sum_{\alpha \geq 0,\ \sum_{i=1}^{d} \alpha_i/n_i \leq 1} b_\alpha \phi_\alpha(\mathbf{x})$$

  - number of terms: J
  - complete polynomials have form $(n, ..., n)$, $\sum_{i=1}^{d} \alpha_i \leq n$
- Anisotropic approximation nodes: $N = \prod_{i=1}^{d} m_i$, $m_i = n_i + 1$

n_1	n_2	n_3	n_4	n_5	n_6	J	N	speedup vs. complete
6	6	6	6	6	6	924	117,649	1
6	**6**	**6**	**4**	**4**	**2**	**267**	**25,725**	**16**
6	4	4	4	2	2	116	7,875	119
6	2	2	2	2	2	42	1,701	1,522

Numerical Dynamic Programming

- Initialization. Choose the approximation grid, $X = \{\mathbf{x}_i : 1 \le i \le N\}$, and choose functional form for $\hat{V}(\mathbf{x}; \mathbf{b})$. Let $\hat{V}(\mathbf{x}; \mathbf{b}^T) = V_T(\mathbf{x})$.
- Iterate through steps 1 and 2 over $t = T - 1, ..., 1, 0$.
  - Step 1. Maximization step: Compute

$$v_i = \max_{a_i \in \mathcal{D}(\mathbf{x}_i, t)} u_t(\mathbf{x}_i, a_i) + \beta \mathbb{E}\{\hat{V}(\mathbf{x}_i^+; \mathbf{b}^{t+1})\},$$

    for each $\mathbf{x}_i \in X$, $1 \le i \le N$.
  - Step 2. Fitting step: compute the $\mathbf{b}^t$ such that $\hat{V}(\mathbf{x}; \mathbf{b}^t)$ approximates $(\mathbf{x}_i, v_i)$ data.

Blue Waters and Parallelization

Today's example

- Approximation nodes in $\mathbf{x} = (k, \mathbf{M}, \mathbf{T}, \zeta, \chi)$ space: 16,129,575 points
- Total number of optimization problems: five billion
- Use Master-Worker approach for each VFI

Another example incorporating tipping points

- Approximation nodes is $\mathbf{x} = (k, \mathbf{M}, \mathbf{T}, \zeta, \chi)$: 1.5×10^9 points
- Total number of optimization problems: 372 billion
- 84K cores
- 8.1 hours (77 core-years)
- Linear scaling
- Each value function iteration uses 12GB in memory

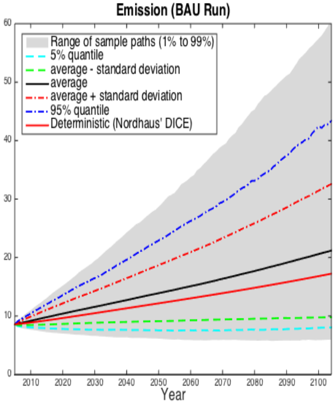
Large Uncertainty from One Case: BAU scenarios

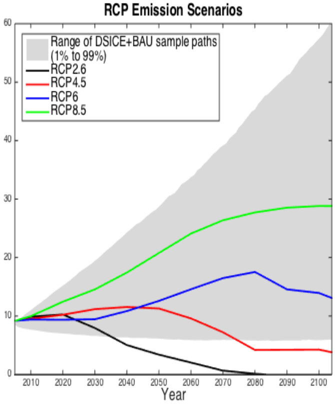
IPCC has promoted the examination of four scenarios

- Supposed to represent range of plausible GHG emission paths
- Used to create input for climate system models

DSICE **with one parameterization**

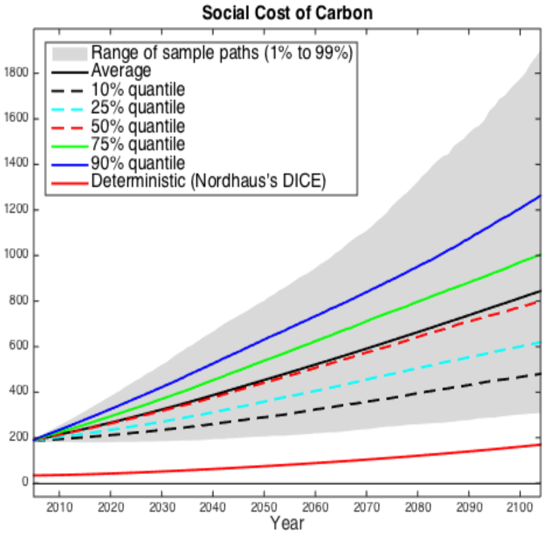
- Produces a probabilistic characterization of GHG emissions
- Shows a range of substantially greater that the IPCC scenarios
- IPCC misses the tail we care about!





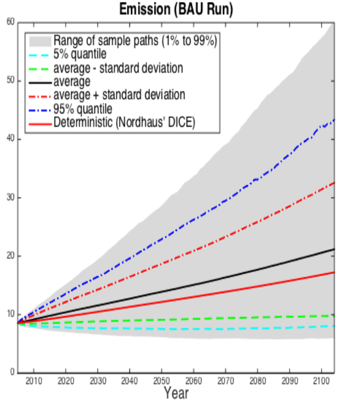
Social Cost of Carbon in DSICE in BAU

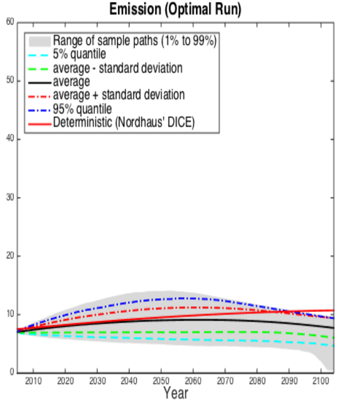
The marginal social cost of carbon could be quite high with significant probability



Emissions: BAU vs Optimal Policy

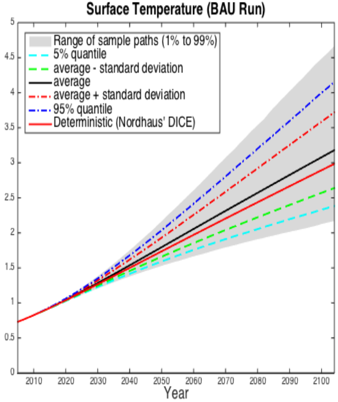
Optimal policy would create substantial reduction in emissions

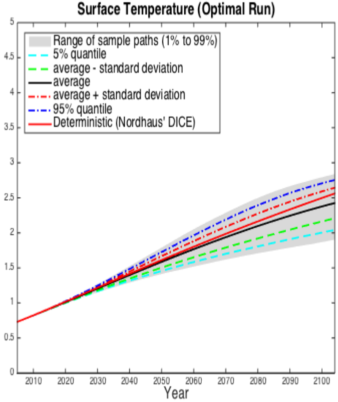




Temperature: BAU vs Optimal Policy

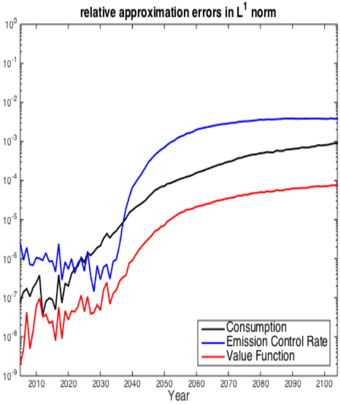
Optimal policy would create substantial reduction in future temperatures

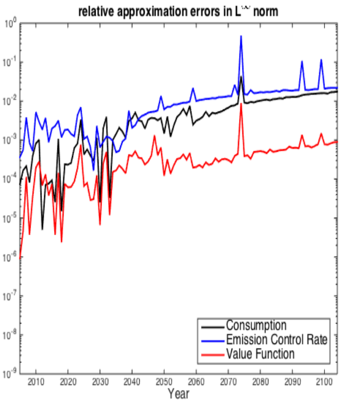




Verification of Results

At each iteration, we verified the accuracy of the approximation by evaluating approximation errors at a random sample of points in the state space

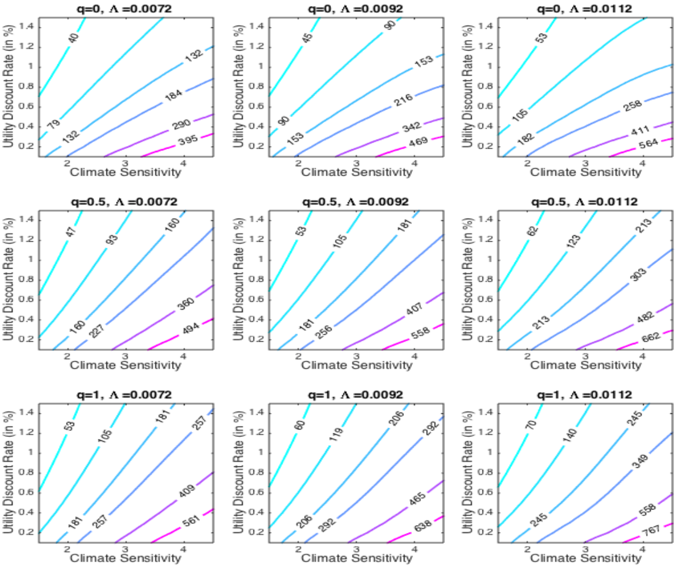




Uncertainty Quantification in DSICE

- Economics must do UQ!!!!!!
  - Some say "We don't know enough to do serious analysis"
  - However, the same people claim to know answers to policy questions
  - Epistemology in economics
    - Remember conclusions of theorems, forget the assumptions
    - Agree with Einstein's advocacy of simple models, but forget "not simpler than necessary"
    - Preference for ad hoc models
  - Goals: unattainable vs. reasonable
    - Parameter and model uncertainty prevents high precision answers
    - UQ helps avoid choosing really stupid policies
- Four uncertain parameter values
  - climate sensitivity
  - damage factor
  - utility discount rate
  - economic growth trend
- Use Smolyak approximation on 4D parameter space, sweeping over Smolyak nodes

Surface Response Function for Uncertainty Quantification



Publications Using Blue Waters

- Lontzek, T.S., Y. Cai, K.L. Judd, and T.M. Lenton (2015). Stochastic integrated assessment of climate tipping points calls for strict climate policy. *Nature Climate Change* 5, 441–444.
- Cai, Y., K.L. Judd, T.M. Lenton, T.S. Lontzek, and D. Narita (2015). Risk to ecosystem services could significantly affect the cost-benefit assessments of climate change policies. *Proceedings of the National Academy of Sciences*, 112(15), 4606–4611.

Working Papers Using Blue Waters

- Cai, Y., K.L. Judd, and T.S. Lontzek (2015). The social cost of carbon with economic and climate risks. Under review in *Journal of Political Economy*, arXiv preprint arXiv:1504.06909.
- Cai, Y., K.L. Judd, and J. Steinbuks (2015). A nonlinear certainty equivalent method for stochastic dynamic problems. Under review in *Quantitative Economics*.
- Yeltekin, S., Y. Cai, and K.L. Judd (2015). Computing equilibria of dynamic games. Under review in *Operations Research*.
- Cai, Y., J. Steinbuks, J.W. Elliott, and T.W. Hertel (2014). The effect of climate and technological uncertainty in crop yields on the optimal path of global land use. The World Bank Policy Research Working Paper 7009, under review in *Journal of Environmental Economics and Management*.
- Cai, Y., K.L. Judd, and T.S. Lontzek (2015). Numerical dynamic programming with error control: an application to climate policy. Submitted to *Operations Research*.

Impact

A July, 2014, White House report:

- "The cost of delaying action to stem climate change"
- Incorporated our paper's conclusion that high SCC can be justified without assuming the possibility of catastrophic events

Extensions

- Multiple Interacting tipping points
  - AMOC, GIS, WAIS, AMAZ, ENSO
  - After one tipping event, other tipping points become more or less likely
- Ag and Forestry - we have 14 continuous states and 2 discrete states
- Multiple sectors
- Learning uncertain parameters
  - climate sensitivity
  - productivity growth
  - damage factor
- These are all multidimensional difference equations in a Banach space of well-behaved functions

Thank you

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Conclusions

Economic analysis of policies require the same scale of computational power as used to solve other complex systems.
Economic problems are different from physics and engineering problems

- Different math
  - Unknown functions are relatively smooth, leading to global spectral methods
  - Unknown functions have high dimension
- Different combination of tasks
  - Parallelism breaks big problems into smaller, compute-intensive nonlinear problems
  - Economics applications use little communication relative to compute effort
  - Many economics applications can use asynchronous parallelization