Monte Carlo Neutrino Transport in Core-Collapse Supernovae

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Core-Collapse Supernovae

- $10^{51}$ erg explosion energy
- $10^{53}$ erg neutrinos released

Neutrinos heat the matter under the shock and drive the explosion.

*But how exactly does this work?*

(Ott 2009)
Neutron Star Mergers

- Neutrinos drive outflows that make heavy elements.
- Neutrinos may drive relativistic jets.

Can computational models match this heuristic?

Metzger and Berger (2012)
What is a neutrino?

\[ \sigma_0 = \frac{4G_F^2 \cos^2(\theta_C)(m_e c^2)^2}{\pi(\hbar c)^4} \approx 1.705 \times 10^{-44} \text{ cm}^2 \]
Boltzmann Equation

\[ \frac{\partial f}{\partial t} + \vec{\Omega} \cdot \vec{\nabla} f = C(f) \]

\( \vec{\Omega} \) = direction

\( C(f) \) = collisional terms

\( f(\vec{x}, \vec{\Omega}, E_\nu, t) = N_\nu/\text{str}/\text{Hz}/\text{cm}^3 \)

7-D integro-differential equation!
Neutrinos are fundamental to supernovae and neutron star mergers.

We need the *right* answer.
A Tale of Two Methods

Two very different methods for solving the same problem:

Discrete Ordinates

Discretize into energy/angle bins.

Monte Carlo

Discretize into particles.

\[
\log_{10}\left(\frac{\rho}{\text{g/cm}^3}\right) r = 1000 \text{ km}
\]

\[\times 10^9\]
Key Challenge: Radiation transport is expensive!

DO is diffusive.

MC is noisy.

(Low resolution used in multi-D)  (Must do this $\sim 10^5$ times.)

Large per-node memory required for MC replication parallelism.
A Tale of Two Methods

Blue Waters is enabling the first verification of multidimensional Boltzmann-level neutrino transport codes.

Discrete Ordinates (dynamical)

Nagakura, Yamada, Sumiyoshi

- 1D: 74k core-hours on Japanese FX
- 2D: 9M core-hours on Japanese K

Monte Carlo (static)

Sedonu (Open Source!)

- 1D: 6k core-hours on BW
- 2D: 768k core-hours on BW

We apply both codes to a snapshot during a supernova simulation.
Monte Carlo vs Discrete Ordinates (1D)

Neutrino Energy Density

Neutrino Direction

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Monte Carlo vs Discrete Ordinates (1D)

**Neutrino Energy Density**

- Neutrino Energy Density $E$ (erg cm$^{-3}$)
- $\nu_e$, $\bar{\nu}_e$, $\nu_x$
- DO, MC

**Neutrino Direction**

- DO, MC
- $r = 35$ km, $r = 68$ km, $r = 169$ km

**Error (%)**

- Error vs Radius (km)
- $r_{sh}$

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Monte Carlo vs Discrete Ordinates (2D)

$P_{r\theta}$ - sensitive metric of multidimensional anisotropy

Good agreement in most difficult metrics.
1. Rewrite Boltzmann equation in terms of angular moments of \( f \)

\[
E := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu)
\]

\[
F^\alpha := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha
\]

\[
P^{\alpha \beta} := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha l^\beta
\]

\[
N^{\alpha \beta \gamma} := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha l^\beta l^\gamma
\]

\[
\frac{dE}{dt} = S_0(E, F^\alpha, P^{\alpha \beta})
\]

\[
\frac{dF^\alpha}{dt} = S_1(E, F^\alpha, P^{\alpha \beta}, N^{\alpha \beta \gamma})
\]

\[
\text{...etc}
\]

(Thorne 1981, Shibata 2011)

2. Make a guess for \( L^{\alpha \beta}(J, H^\alpha) \) and \( N^{\alpha \beta \gamma}(J, H^\alpha) \)

3. Evolve \( J \) and \( H^\alpha \) only
The Third Wheel: Approximate Moment Methods

How good is the approximation?

Monte Carlo and Discrete Ord. can make approximate transport better!
An Aside: Relativistic Random Walk Approximation

1) Use analytic solution to find travel time.

2) Transform out of the comoving frame.
Take Away

- **Blue Waters Fellowship** is funding development and *rigorous* verification of new methods.

- **Blue Waters hardware** is optimal for MC radiation transport.

- **Blue Waters support** has been instrumental in aiding optimization and implementation ideas.

- **Open-source code** and **open data** aid verification efforts community-wide.

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